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THE

PHYSICAL REVIEW.

LATENT HEAT OF FUSION AS THE ENERGY OF MOLECULAR ROTATIONS.

By Kôtarô Honda.

IN the theory of specific heat of a solid, it is usually assumed that each molecule (or atom) of a substance vibrates about its mean position with three degrees of freedom. Since the motion of each molecule is a vibration, but not a uniform translation, its energy is just twice the mean kinetic energy during a complete period of vibration. Hence the energy belonging to each degree of freedom is given by

$$E_T = \frac{h\nu}{e^{h\nu/kT} - 1},$$

if we assume the quantum hypothesis. T is the absolute temperature, k the gas-constant referred to one molecule, h the universal constant and ν the frequency of vibration.

We may also with good reason suppose that the above mode of rectilinear vibration is associated with small rotational vibrations of the molecules (or atoms) about their centers of mass, probably with a common period of vibration. At ordinary temperatures, this mode of rotational vibration is very small; but as the temperature becomes higher, it increases at a steady rate. The fact that at very high temperatures, the specific heat experimentally found is in many cases greater than the theoretical value, 5.96, indicates the presence of this rotational vibration. In approaching the melting point of the substance, the amplitude of this vibration attains such a value that the rotational vibration at last becomes a continuous rotation. In the case of a cubic crystal, it is possible that this limiting half-amplitude is a little less than $\pi/4$. As soon as the rotational vibration is changed into a continuous rotation, the regular spacing of the molecules breaks down, disorder beginning to take place, and therefore the rotation becomes independent of the translation. For, since the molecules of a solid may be regarded as sending out their tubes of force in a few definite directions, the regular spacing of the

molecules is only possible for their definite orientations. The molecules thus set free have now two or three degrees of freedom for rotation; the angular velocity of each component rotation increases to such an extent that its energy becomes equal to that corresponding to one degree of freedom, that is, $1/2\ E_{T_m}$, T_m being the melting point. If all the molecules undergo the above change, the fusion is completed and the crystal changed to a liquid.

If there are N molecules in one gram-molecule, the total energy of rotation received by the molecules is $Ne_{T_m}(= E_{T_m})$ or $3/2\ Ne_{T_m}$ $(= 3/2\ E_{T_m})$ according as the liquid molecules have the shape of a body of revolution or not. Since the molecular distances in solid and liquid states do not differ much, the rectilinear motion of the liquid molecules is also vibratory, its amplitude remaining the same. Hence the translational energy of each liquid molecule corresponding to one degree of freedom must be equal to e_T, that is, the same as that in the solid state. In general, the change of volume during melting is small, so that the energy required for doing the internal and external work is also small; we denote it by Q. If λ be the latent heat of fusion of a substance and w its molecular (or atomic) weight, we have the relation

$$w\lambda = nE_{T_m} + Q,$$

where $n = 1$ or $3/2$. If we neglect Q in this relation, we have simply

$$w\lambda = nE_{T_m} \quad \text{or} \quad \frac{w\lambda}{T_m} = \frac{nE_{T_m}}{T_m};$$

E_{T_m}/T_m has a value a little less than $R = 1.98$, the gas-constant referred to one gram-molecule, the calorie being the unit of heat.

In order to see whether in the case of elements, the above simple relation is actually satisfied or not, we must calculate E_{T_m}. For this purpose, it is necessary to know the frequency ν proper to the elements. This value can be calculated by one of the formulæ by Einstein,[1] Lindemann,[2] Alterthun,[3] and Grüneisen.[4] The values of ν for the same element as calculated by the different formulæ do not exactly coincide with each other; but at high temperatures, the value of E_T varies very little with a small change in the value of ν, and therefore the disagreement among the values of ν does not much affect the values of E_{T_m}. In our case, E_{T_m} was calculated from the values of ν given by Lindemann's formula

$$\nu = k\sqrt{\frac{T_m}{wV^{2/3}}}$$

[1] Ann. der Phys., 34, 1911, 170.
[2] Phy. Zeitschr., 11, 1910, 609.
[3] Deutsch. Phys. Gesell. Verh., 15, 1913, 25, 65.
[4] Ann. der Phys., 39, 1912, 257.

where V is the atomic volume and 3.08×10^{12} taken as the value of k. The following table contains the melting points T_m, the atomic heat of fusion $w\lambda$ and their ratios $w\lambda/T_m$ for different elements, as given in Landolt and Börnstein's table, together with the values of v and nE_{T_m}, the values of n being properly chosen.

Group I.

Elements.	v.	T_m.	$w\lambda$.	$w\lambda/T_m$.	nE_{T_m}/T_m.	n.
Pt	4.75×10^{12}	2028° K	5360 Cal.	2.64	2.67	3/2
Pd	6.15	1818	3860	2.11	1.80	1
Fe	9.09	1773	3348	1.89	1.75	1
Ni	8.87	1724	2710	1.57	1.75	1
Cu	7.39	1356	2650	1.96	1.73	1
Ag	4.29	1234	2270	1.84	1.82	1
Al	8.32	930	2160	2.33	2.37	3/2
Zn	4.78	692	1840	2.66	2.49	3/2
Pb	1.99	600	1118	1.86	1.83	1
Cd	3.01	594	1540	2.59	2.64	3/2
Tl	2.01	560	1470	2.60	2.73	3/2
S_2	4.26	392	660	1.70	1.51	1
Na	4.30	390	730	1.87	1.49	1
K	2.53	331	610	1.84	1.64	1
P_4	3.83	317	584	1.84	1.46	1
Hg	1.38	234	562	2.40	2.58	3/2

Group II.

Bi	1.79×10^{12}	544° K	2580 Cal.	4.74	2.76	3/2
Sn	2.51	505	1667	3.30	2.64	3/2
I	1.82	389	1488	3.83	2.64	3/2
Ga	2.81	303	1339	4.39	2.34	3/2
Br	1.38	266	1280	4.82	2.61	3/2
Cl	——	170	814	4.79	——	3/2

From the above tables, we see that for the elements in group I., the above relation is well satisfied, though the melting points of these elements vary from 2028° K. to 234° K. Here it is to be remarked that the values of λ are not accurately known and usually admit of an error amounting to 10 per cent. Considering these circumstances, we may conclude that for these elements, the value of Q is negligibly small.

The elements in group II. have a much greater value for the ratio $w\lambda/T_m$ than for the values of nE_{T_m}/T_m. It is however to be noticed that the two elements bismuth and gallium appear to undergo a transformation during melting; because the electric resistance of these two elements diminishes during melting,—an abnormal phenomenon. Hence, the latent heat of fusion is probably increased by the heat of this transformation. A similar remark will also apply to tin, the sign of the mag-

netic susceptibility of which changes abnormally during melting. It is well known that the halogen elements have a large specific heat; this is probably the main cause of the large deviation of the above ratio from the values of nE_{T_m}/T_m, the value of Q being considerably large.

From the above result, it is to be concluded that unless some transformation takes place in a substance during fusion, or its specific heat is abnormally large, the latent heat of fusion consists of the energy of rotation of the molecules, gained during fusion.

As to the compounds, a similar relation holds good; but here the ratio $w\lambda/T_m$ is generally much greater than that in the case of the elements, the values lying between 2 and 10. Some results of my calculation are given in the following tables:

Group I.

Compounds.	V.	T_m.	$w\lambda$.	$w\lambda/T_m$.	nE_{T_m}/T_m.	n.
KOH	3.26×10^{12}	633° K	1610 Cal.	2.54	2.61	3/2
NaOH	4.87	691	1600	2.33	2.49	3/2
RbOH	2.21	574	1620	2.82	2.73	3/2
CsOH	1.71	545	1610	2.95	2.73	3/2
AgBr	1.92	703	2370	3.38	2.79	3/2
Group II.						
KF	4.67×10^{12}	1133° K	6270 Cal.	5.53	4.50	5/2
NaCl	4.50	1077	7220	6.71	4.45	5/2
KCl	3.41	1045	6410	6.13	4.60	5/2
TlBr	2.37	733	3610	5.93	4.55	5/2
AgCl	2.34	728	3050	4.15	4.55	5/2
TlCl	1.62	700	3980	5.69	4.65	5/2
$BaCl_2$	1.66	1232	5800	4.70	4.80	5/2
$CaCl_2$	3.17	1047	6060	5.78	4.60	5/2
$PbCl_2$	1.34	764	5150	6.67	4.75	5/2
$PbBr_2$	1.16	761	3650	4.80	4.75	5/2

For the compounds of group I., if we assume three degrees of freedom for rotation, the values of $w\lambda/T_m$ nearly coincide with those of nE_{T_m}/T_m. For the halogen compound in the same group, the value of $w\lambda/T_m$ is considerably larger. The diatomic compounds may have five degrees of freedom for rotation—3 as a whole molecule and 1 for each atom; the triatomic or multiatomic compounds may have more degrees of freedom than five. For the elements in group II., we have calculated nE_{T_m}/T_m for $n = 5/2$. As in the case of elements, the ratio $w\lambda/T_m$ for halogen compounds is generally much greater than the corresponding values of nE_{T_m}/T_m. For the compounds, there is, however, some ambiguity in choosing the number of degrees of freedom gained during fusion. Hence

the case of the compounds cannot be considered as a verification of the above theory.

Next, we shall consider the specific heat of a liquid. If the temperature of a liquid is high, E_T is nearly equal to RT, where R is the gas-constant, and the total content of heat energy possessed by a liquid per gram molecule is

$$3RT + nRT = 4RT \text{ or } \tfrac{9}{2} RT$$

for an element. Hence the specific heat at constant volume is

$$C_v' = 4R \text{ or } \tfrac{9}{2} R = 7.92 \text{ or } 8.91.$$

According to the first law of thermodynamics, we have, at the melting point of a substance,

$$\frac{d(w\lambda)}{dT_m} = C_p' - C_p,$$

where Cp' and Cp are the specific heats of liquid and solid states at constant pressure, respectively. But, as we have seen above, we have for elements

$$w\lambda = nE_{T_m}, \; n = 1 \text{ or } 3/2.$$

$$C_p' = C_p + n\frac{dE_{T_m}}{dT_m}.$$

Since, at melting point, dE_{T_m}/dT_m has a value nearly equal to R, the above relation may be written

$$C_p' = C_p + nR = C_p + 1.98 \text{ or } C_p + 2.97.$$

Generally speaking, these results agree with the observed facts.

As a further confirmation of the above view, we may also add the following facts. It is well known that during gradual melting of an amorphous substance, no appreciable amount of latent heat is absorbed. Since an amorphous substance is to be considered as an extremely viscous fluid, its molecules in a solid state already possess freedom of rotation, so that for liquefaction no latent heat is required.

It is also generally admitted that the molecules of a gas or a liquid exert their action according to the law of central force, that is, the molecular force radiates from its center uniformly in all directions. For explaining a crystalline structure, it is, however, necessary to assume that each molecule sends out its tubes of force only in a few definite directions so as to form a given arrangement of the molecules in the space-lattice. What change will then occur in the molecules during fusion? According to our view, no change generally takes place in the molecules; but during melting, the molecules acquire rotational motion, whose axes are continuously changing from one instant to another, so that the mean

action of the molecular force is equivalent to the force equally distributed in all directions.

Lastly it should be noticed that according to our view, even the molecules of a monoatomic element make, in a liquid state, rotational motion with two or three degrees of freedom, the energy of rotation increasing with its temperature. During vaporization, the molecules start with the initial energy of rotation; but this energy does not increase with the rise of temperature, as is the case with diatomic or polyatomic gases. For, in the gaseous state, in which the mutual distances of the molecules are very large as compared with those in a solid or liquid state, the molecules of a monoatomic element are considered to behave with respect to their mutual impacts like small spherical balls and therefore their impact cannot cause any further increase of the energy of rotational motion. Hence, the energy content of a monatomic gas may be given by

$$Q = \tfrac{3}{2} RT + nR\theta,$$

where θ is the temperature of vaporization; its specific heat C_v is therefore

$$C_v = \frac{3R}{2} = 2.97,$$

which agrees with the observed value.

THE PHYSICAL CHARACTERISTICS OF X-RAY FLUORESCENT INTENSIFYING SCREENS.[1]

By Millard B. Hodgson.

THE use of fluorescent screens for the intensification of exposure is of widespread practice and of considerable importance in practical x-ray photography or roentgenology. Lately, in the work of Hull, St. John and others, the fluorescent screen has been employed to reduce the lengthy exposures of such x-ray spectroscopic investigations that are dependent on the photographic plate for record.

In spite of the general use of these screens in quite a variety of work, very little has been done in the study of the fundamental laws of such a system. Shearer,[2] Edwards,[3] Baker[4] and others, however, have contributed excellent papers to the general fund of knowledge from a strictly utilitarian standpoint. Recently Sheppard[5] advanced some interesting theories regarding conditions for the fluorescent state.

It is the purpose of the present paper to outline briefly the results of some experiments introductory to a general study of fluorescence to x-rays.

In dealing with fluorescent phenomena dependent on an x-ray tube for excitation, there are two types to be considered, both of which may be utilized for photographic intensification. The first type is that of true characteristic radiation, the second vultra-violet and visible fluorescent radiation. The true characteristic radiations of all the elements which are feasible to use are within that range of frequencies usually termed x-rays. Hence, the laws pertaining to high frequency radiation govern the photographic use of screens dependent on this principle. In the case of fluorescent emission of ordinary light, the laws of ordinary optics apply. The emitted radiation also seems no longer fundamentally dependent on the atom alone but on the space lattice pattern of the crystal system of the particular fluorescent solid used.

Of these two types of intensifying screens the second has proven the far more efficient in practical usage.

[1] Communication No. 67 from the Research Laboratory of the Eastman Kodak Company.

[2] J. S. Shearer, American Journal of Roentgenology, February, 1914.

[3] H. T. Edwards, American Journal of Roentgenology, December, 1916.

[4] H. Thorne Baker, Journal of the Roentgen Society (British), October, 1917.

[5] S. E. Sheppard, Illuminating Engineer (London), June, 1917.

Herschel, Stokes, Wood, Wiedeman, and Nichols and Merritt have contributed chiefly to the general subject of fluorescence within the ordinary range of frequencies usually called light. Of these Nichols and Merritt have perhaps done the most complete quantitative work.

The investigation of the phenomena of fluorescence of materials to x-rays, however, has not been as yet thoroughly investigated.

Of the materials which fluoresce to x-rays in the range of frequencies from the ultra-violet to the red, there are only a few which can be used efficiently for photographic intensification. All of these substances must be in the crystalline state. Barium-platino-cyanide, barium salicylate, calcium tungstate, molybdenum tungstate, magnesium tungstate and some double tungstates of these metals fluoresce to a greater or less extent to x-rays and the radiation is more or less active photographically. Of these, crystalline calcium tungstate is by far the best, with present photographic x-ray materials. The salt is usually powdered and coated with a suitable binder on a support of some material of slight x-ray absorption, such as cardboard or celluloid. This screen is then placed in contact with the photographic surface and exposure made through either the screen or the photographic plate or film.

The efficiency of any radiator as a source of photographic stimulation depends primarily on the comparative spectral distribution of the energy of the radiator and the spectral sensibility of the particular photographic plates used. While these relations have not been determined as yet on an equal energy basis for x-ray materials, qualitative analyses have been made.

In Fig. 1 are shown the spectral sensibilities to white light of an (*a*) x-ray plate, (*b*) a fast ordinary plate, (*c*) an orthochromatic plate and (*d*) a panchromatic plate.

The fluorescent spectra of calcium tungstate have been studied under the following conditions. The screens used were commercial types. Spectra were obtained using a Hilger quartz spectrograph on Seed X-Ray, Seed 30 and Wratten Panchromatic Plates. The illustrations shown were made on Seed 30; the spectral distribution of photographic energy in the fluorescence does not, however, extend appreciably further toward the red than shown on these photographs. The screen was placed immediately in front of the slit and the x-ray tube in front of the screen as shown in the diagram (Fig. 2). A Coolidge tube of medium focus was used, the length of exposures averaging 1,000 milliampere minutes at 8 in. distance from the target to the screen. The photographic plate was carefully shielded from stray radiation during the long exposures, which in some cases were for 24 hours.

a

b

c

d

FIG. 1.

40 K.V.

60 K.V.

80 K.V.

FIG. 3.

M. B. HODGSON.

Spectrograms were made with the tube operating at 40 K.V., 60 K.V. and 80 K.V. (R.M.S.). These are illustrated in Fig. 3, losing, however, by reproduction. It will be noticed that there is a rather abrupt jump from a continuous or at least a broad-banded spectrum to a relatively narrow band spectrum at 80 K.V. It is a significant fact that at 80 K.V. the α and β lines of the K series of tungsten begin to be noticeable in the primary x-ray spectrum of the tube used. Below this voltage, the variation of intensity of the tungsten spectrum with wave-length is a smooth curve, being practically continuous except for the "L" lines which however seem to exert but slight influence on the fluorescence of calcium tungstate. This suggests some peculiar resonance phenomena taking place when the electronic orbits are selectively "stirred" by the incident radiation reaching a certain critical frequency.

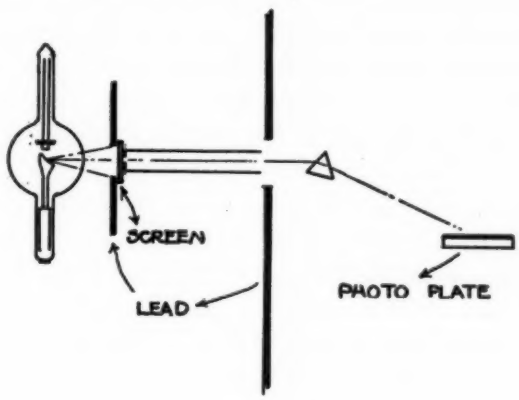

Fig. 2.

The fluorescence is certainly primarily dependent on the following factors: (1) selective absorption of the elements of the crystalline compound present in the intensifying screen, (2) the frequency of the exciting radiation, and (3) the crystal structure of the particular calcium tungstate used. The x-ray analysis of the crystal structure of calcium tungstate will probably throw a great deal of light on the subject.

A comparison of the curves in Fig. 2 with the spectra in Fig. 3 will enable one to pick out the plate most efficient for use with the screen. However, in a final consideration of efficiency the sensitiveness of the material to the direct x-ray beam must be given weight. It has been found that in a negative made from a screen system, about 20 per cent. of the exposure is due to the absorbed energy of the direct rays, the remaining 80 per cent. being supplied by the screen fluorescence. Hence to decide the maximum efficiency in the case where the ratios of screen

fluorescence and plate spectral sensibility are the same for two photographic materials, their relative sensitiveness to x-rays alone should be considered.

As to technique of using a screen, the best practice is to expose through the support of the emulsion which is in contact with the surface of the fluorescing screen. Hence it follows that the support must be one of minimum opacity to x-rays. In the case of two emulsions of equal sensitiveness, one on glass and the other on transparent, flexible base (film), the latter support should be used, being more transparent to x-rays.

This support has the further advantage of permitting bending which is to be desired in some spectroscopic experimental work. In the precise determinations of high frequency wave-lengths, however, it will be the better policy to use emuslion coated on plane surfaced glass. Computation shows that the error involved in a wave-length determination on film, due to shrinkage of the support, may be as high as one per cent. of the wave-length.

As may be seen from the spectra in Fig. 2, the output of radiant energy photographically suitable increases with voltage. It also increases with current to a saturation value, dependent on the particular screen used. The limiting voltage, however, which it is permissible to use in practical radiography, is governed by the absorption of the object radiographed. That is, in the case of the average picture of body parts, in order to absorb enough of the incident x-ray beam to differentiate fine detail the penetrating ability of the beam should be of a certain minimum value, of necessity limiting the tube voltage.

In spectroscopic investigation, for instance, in crystal structure determination by the Hull method, penetration is of no great consequence. The resolving power of the system, however, greatly decreases with increased voltage, due to the penetration and scatter of the shorter wave-lengths in the screen.

In the recording of the shorter wave-lengths of x-rays such as used in the radiography of castings or welds, in view of the great penetrating power of the rays, the photographic plate fails to absorb sufficient energy to give a picture permitting of clear interpretation. In these cases the usual calcium tungstate screen, while fluorescing, causes a grainy deposit. The writer in trying to shorten such exposures has tried using the characteristic radiation from metal screens.

Silver, copper, lead, tungsten and platinum have been used. Of the materials tried silver and platinum were the most practical and efficient. A sheet of silver .2 mm. thick gave an intensification under the standard

conditions used of 100 per cent. Platinum, used economically in the form of a cathodically sputtered plane mirror, of a film thickness of less than .001 mm. gave an intensification of 20 per cent. These per cent. differences while seemingly small in average photographic practice where the exposure is a matter of seconds, become relatively large when one considers running an x-ray tube minutes and hours for a single exposure. The use of such screens does not involve any increase in graininess.

Summary.

1. The fluorescence of various materials to x-rays has been discussed from the point of view of photographic efficiency.

2. The spectral distribution of the fluorescence from calcium tungstate has been qualitatively determined and has been found to extend further into the ultra-violet as the voltage applied to the exciting tube increases.

3. The photographic efficiency of the characteristic radiation from silver, tungsten, platinum and lead has been approximately determined for certain photographic materials and found to be of practical value.

Research Laboratory,
Eastman Kodak Company,
May 6, 1918.

THE SPECIFIC HEAT OF PLATINUM AT HIGH TEMPERATURES.

BY WALTER P. WHITE.

THE specific heats of metals at high temperatures are of special theoretic importance on account of the light they may throw on electronic theories, and among the metals platinum, for fairly evident reasons, is especially well adapted for an investigation of this kind. The present paper offers some new values of the specific heat of very pure platinum from 100° to 1300°.

The apparatus and methods are the same as are to be described in presenting some work on the specific heats of certain silicates[1]—in fact, the two investigations were carried out together. Since the charge of platinum had about one third the heat capacity of the usual charge of silicate, the work with it was the measurement of a quantity one third as great, so that a precision a third as good, or to 3 per mille, would have seemed satisfactory, but in all but two cases this precision was exceeded. There is of course no reason to suspect a systematic error any greater than in the case of the silicates. Numerous and varied cross checks indicated that with them the systematic error did not reach 3 per mille in the worst instances. It is therefore probable, though of course not certain, that the discrepancy of .003 in the platinum values is the greatest error present.

Two samples of platinum were used. One was of Heraeus' purest thermoelement wire, drawn to 2 mm. diameter, coiled to a convenient bulk, and annealed in an oxidizing flame at a temperature of about 800°. It was also abundantly annealed at 700° in the course of the first specific heat determinations. This valuable wire, to avoid possibility of contamination, was not used above 700°. Above that temperature a lower grade of platinum was used, most of which was also from Heraeus and guaranteed not to contain over 0.005 of iridium. At 700° this wire gave a result 0.00012 in 0.03, or about 3 per mille, higher than the purest wire. It seemed best to diminish by this amount all the results with the lower grade wire, and to consider these as corrected for the variation from purity. The total value of this correction is only the maximum error of the observations. Moreover, it *lowers* the specific heat—a point of importance in comparing with some results obtained by others.

[1] Silicate Specific Heats, Second Series, Walter P. White, Am. J. Sci., 47, 1, 1919.

Table I. gives the results. The differences in column 4 indicate a linear variation of the interval specific heat to well within the small experimental error, except between 100° and 300°, where the difference is 0.0008 for the 200 degrees. The very accurate results of Gaede[1] and Schlett[2] show that below 100° the variation reaches 0.0000065 calories per gram per degree (or 0.0013 for 200°) with a tendency to diminish; it may therefore very probably be .0008 below 300°. The present results at 100° certainly show very satisfactory precision, and are also much less to be suspected of systematic error than those at the higher tempera-

TABLE I.

Specific Heat of Platinum.

	Interval Heat.	Mean.	Difference.
20° – 100°	(.03167)		
	87		
	88	.03186	
	84		.00085
20° – 300°	.03273		
	68		
	62	.03271	
	78		
	72		0.0062
20° – 500°	.03333		
	30	.03333	
	36		.00066
20° – 700°	.03398		
	.03399	.03399	
	399		
	.....		
	3397		
	3400		.00061
20° – 900°	.03462		
	58		
	60	.03460	
	61		.00063
20° – 1100°	.03524		
	22	.03523	
			.00069
20° – 1300°	.03592		
	93	.03592	

tures. The slight increase in the difference between 1100° and 1300° does not necessarily indicate any real departure from the linear relation. In my earlier results (1909) there was a *decrease* from 1300° to 1500°, in which[3] Koeningsberger found confirmation of a certain theory he had

[1] Quoted by Magnus in the reference given later.

[2] W. Schlett, Ann. d. Phys., 26, 201, 1908.

[3] Ueber den Anteil der freien Elektronen an der spezifische Wärme. J. Koenigsberger. Ber. Physik. Ges., 14, 278, 1912.

as to the variation with temperature of the number of electrons in metals. I have always suspected that the decrease was only experimental error, though I apparently did not bring that out clearly in the 1909 paper.

The results at 300° and higher are well represented by the formula:

$$.03175 + 0.0000032\,\theta, \tag{1}$$

which gives for the true specific heat[1]

$$.031686 + 0.000064\,\theta. \tag{2}$$

The resulting value for the true atomic heat is:

$$6.185 + 0.00125\,\theta. \tag{3}$$

This gives: at 500°, 6.81; at 1300°, 7.81. These values of course are for constant pressure. Values for constant volume can be obtained by the formula given in the silicate paper referred to:

$$C_p - C_v = \frac{.02345(3\alpha)^2\theta}{Kd}, \tag{4}$$

where d is density, K compressibility (Kg per cm.^{2}), 3α the *cubic* expansion coefficient, and the result is in calories per gram.

If we take compressibilities by linear extrapolation from Grüneisen's data,[2] a process which gives, at 500°, 900° and 1300°, .432, .465, and .498, $\times 10^{-6}$, and derive both volume and expansion from the Holborn and Day formula: vol. $= (26.6\theta + .004\theta^2)\ 10^{-6}$, expansion coefficient $= (26.6 + .008\theta)\ 10^{-6}$, we get:

		C_v	A_v	ΔC_v
At	500°	.03303	6.447	
				.00118
	900°	.03421	6.678	
				.00086
	1300°	.03506	6.844	

A_v is clearly above the kinetic theory value, 5.96, and increases steadily at a rate somewhat less than 9 per mille per 100°. The increase is not linear, but the variation from linearity is not certainly greater than the possible errors. If the compressibility at high temperatures increases more than in proportion to the temperature, as seems not improbable from Grüneisen's results for lower melting metals, the variation might be considerably reduced. The data, also, are evidently not inconsistent with larger values at the higher temperatures. There is no reason to suppose that the specific heat *at constant pressure* really varies linearly, since

[1] The first term differs from that in (1) because (1) is for intervals beginning with 20° not 0°.

[2] E. Grüneisen, Ann. der Physik, 23, 1264, 1910.

the contribution of expansion work to the specific heat does not. The only reasons for the linear formula are its simplicity, and the danger that a more elaborate expression might give an exaggerated idea of the supposed accuracy of the data.

Comparisons with Previous Work.

The results of Pouillet (1836) and Violle (1877), important and excellent in their day, differ materially from those obtained with modern resources, and do not seem to be longer of practical value. Plato[1] appears to have been the first to get results which still seem satisfactory. He found .03369 and .03430 respectively for the intervals from room temperature to 600° and 750°. My own earlier results,[2] published in 1909, gave $0.03198 + 0.0000034\theta$, from 0° to θ, which is about 6 per mille too high.[3] Corbino[4] employed a very interesting method, based on the time required to heat a platinum wire by an electric current, and Fabaro[5] confirmed his values by a different method embodying the same principle. One feature of Corbino's presentation is rather curious. His method gives only the *change* of specific heat with temperature. He quoted my 1909 result at 500° in order to assign absolute values for his own, but made no reference to any other of my results, or even to the fact that I had any others. He finds that C_v increases 14 per mille per 100°, which brings his results into very good agreement with these here, if we consider that he only estimates the precision of any value as 1 per cent. A. Magnus,[6] using a dropping method, measured from 500° to 900°, that is, through one third the range of the present work, with a mean *accidental* error of 2 per mille, getting $0.031590 + 0.00000292340\theta$ for the interval heat. These coefficients are respectively about 4 per mille and 6 per cent. smaller than those contained in this paper. He considered his results good to 5 per mille.

Up to 100° determinations of the specific heat of platinum are more numerous. A good account of them is given by K. Schulz in the Fortschritte der Mineralogie for 1912 and for 1913.

[1] Erstarrungserscheinungen an anorganischen Salzen und Salzgemischen. W. Plato, Z. physik. Chem., 55, 735, 1906.

[2] Specific Heats of Silicates and Platinum. W. P. White. Am. J. Sci., 28, 342, 1909.

[3] These determinations were only estimated as good to 5 per mille. They were made because we wanted the platinum values for use as a correction in silicate work with platinum containers. The use made of these results by others led us to think that more accurate ones would be acceptable.

[4] Theremc-kalorimetrische Untersuchungen am Platin. Physik. Z., 14, 915, 1913.

[5] Sul Calore Specifico del Platino a Elevati temperature. L. Fabaro, Nuovo Cim., 9, 123, 1915.

[6] Die Spezifische Wärme des Platins und des Diamenten bei hohen Temperaturen, Ann. d. Physik, 48, 997, 1915.

The variations between different observers are very wide; values run from .03147 to .0328 for the interval 0° to 100°. These divergences may be in part accounted for by the impurity of the platinum used by some of the earlier workers, and in part, by differences in the physical condition of the metal. Schlett[1] finds very curious differences of this kind, though his experiments do not seem complete enough to lead to any general conclusions, except that there is a general tendency for the specific heat to vary with the specific volume. Gaede's probably very accurate results[2] give .03175 for the interval 20°–100°, which is below the present result of .03186, but scarcely differs from it by more than the observational errors. Battelli and Stracciati (1895) and Wigand (1907) are about a per cent. higher than the result given here. (Wigand gives two results differing a per cent. from each other.) Nearly everyone else is higher still. With the earlier workers this may sometimes be the result of impurity, but Jaeger and Diesselhorst, who had pure platinum, get two results respectively 2 per cent. and 3 per cent. above Gaede's. One may suspect that their apparatus, designed for different work, was not very well adapted for determining specific heats.

In 1912 Koenigsberger,[3] using my 1909 results, and also those of Kunz,[4] calculated that the atomic heat at constant volume of platinum is above the Dulong and Petit value, and continues to increase with temperature. He very properly considered the question whether for a conclusion so important the data were sufficiently assured. The results which have since accumulated, for other metals as well as platinum, seem to leave no further doubt.

Corbino[5] also finds that tungsten shows an increase of specific heat above the theoretical 5.96, though his actual values differ considerably from the probably better ones of Worthing.[6] Worthing finds, between 1,200 abs. and 2,400 abs. generally lower absolute values than those of platinum, but a higher rate of increase, about 13 per mille per 100°.

SUMMARY.

The specific heat of platinum has been redetermined from 100° to 1300° with a precision estimated to be better than .3 per mille and with

[1] Loc. cit.

[2] Wolfgang Gaede, Ueber die Aenderung der spezifischen Waerme der Metalle mit der Temperature, Physik. Z., 4, 105, 1902; Fortschr. Physik, 373.

[3] Loc. cit., p. 279.

[4] L. Kunz, Ann. d. Physik, 14, 309, 1904.

[5] Phys. Zeits., 13, 381, 1912.

[6] Atomic Heats of Tungsten and of Carbon at Incandescent Temperatures. A. G. Worthing, Jour. Franklin Inst., 185, 707, 1918; Phys. Rev., 12, 199, 1918.

very satisfactory agreement with the results of Gaede at 100° and with those of Plato, Corbino, Magnus, and Fabaro at higher temperatures. Most published results below 100° appear to be 1 per cent. or more too high. The atomic heat of platinum at constant volume is, from 100° up, above the value, 5.96, indicated by Dulong and Petit's law, and by the accepted kinetic theories of the solid state, and also increases regularly to 1300°, and probably beyond that.

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THE PHOTOGRAPHIC STUDY OF IMPACT AT MINIMAL VELOCITIES.

BY C. V. RAMAN.

1. INTRODUCTION.

HERTZ'S well-known theory[1] treats the phenomena occurring during elastic impact practically as statical effects; the local compression at the place of contact is regarded as gradually produced and subsiding completely by reversal of the process by which it is produced. In the particular case of two solid spheres impinging on each other, the theory finds its own justification by showing that the duration of impact is large compared with the gravest period of vibration of either body. The results indicated by Hertz's theory for the duration of impact, for the radius of the area of contact, and for the approach of the centers of the spheres have been compared with those found in experiment by various investigators among whom may be mentioned Schneebeli, Hamburger,[2] and more recently, Lafay,[3] Kennelly and Northrup.[4] Hertz's theory is incomplete in that it entirely ignores the dissipation of energy that occurs during collision. Recently, however, Lord Rayleigh[5] has extended Hertz's work by investigating the circumstances of the first appearance of vibrations in the case of two impinging spheres, and has shown that the proportion of translational energy transformed into energy of vibration is negligibly small over a considerable range of velocities. The dissipation of energy that occurs in the impact of spheres has thus to be assigned principally to two causes, (*a*) the stresses occurring during impact exceeding the limits within which elastic recovery is immediate and perfect, and (*b*) the production of sound. The second of these causes has been recently investigated at Calcutta by Banerji,[6] who has shown that the sound waves produced by the impact of spheres are practically

[1] Hertz's Miscellaneous Papers, English edition, p. 146, and Love's Treatise on Elasticity, 2d edition, p. 195.

[2] Schneebeli, Rep. d. Phys., Bd. 22, 1886, and Hamburger, Tageblatt d. Nat. Vers. in Wiesbaden, 1887.

[3] Lafay, Comptes Rendus, p. 525, 1900, and Anna. Chim. Phys., p. 241, 1901.

[4] Kennelly and Northrup, Frank. Inst. Journ., July, 1911.

[5] Rayleigh, Phil. Mag., Vol. XI., p. 283, 1906.

[6] Banerji, Phil. Mag., July, 1916, and January, 1918.

due to the impulse communicated to the atmosphere by the sudden reversal of the motion of the spheres. The energy thus lost may be evaluated mathematically, and being directly proportional to the density of the atmosphere, can be made as small as desired by sufficiently reducing the pressure. If we leave it out of consideration, the dissipation of energy has to be ascribed entirely to failure of perfect elasticity during impact. According to Hertz, if two steel spheres 2.5 cm. radius meet with a relative velocity of only 1 cm. per second, the maximum pressure developed during impact is 7,300 kilogrammes per sq. cm. Stresses of this order of magnitude would, in the case of most materials overstep the elastic limits and result in dissipation of energy. If the substance of the spheres be plastic, the energy of collision may even be dissipated entirely in the production of deformation. This is illustrated by some recent interesting experiments by Pochettino[1] who studied the elasticity, the viscosity, and the coefficient of restitution of pitch at different temperatures. The coefficient of restitution was 0.92 at 10° C., fell to 0.44 at 28.1° C., to 0.12 at 31° C., and was zero at 32° C.

The question naturally arises whether the coefficient of restitution can be connected in some definite manner with the magnitude of the stresses produced by impact. According to Hertz's theory, the maximum pressure developed per unit area is proportional to the two-fifths power of the velocity before impact, and is independent of the size of the balls. Accordingly, by reducing the velocity of impact sufficiently, it should be possible to keep the stresses developed below any definite value which may be assigned as the elastic limit, and the coefficient of restitution for such velocities should approach unity for all solids. That the coefficient becomes smaller with increasing velocity of impact has indeed been shown by Hodgkinson,[2] and more recently also by Vincent.[3] In view, however, of the bearing of the subject on the theory of impact, it appeared worth while to make a more detailed examination, and to push the observations to the smallest velocities up to which it is possible to obtain regular results. In order to make reliable measurements at such small velocities, it was decided to use the photographic method to record the relative motions of the spheres immediately before and after impact, and to determine the coefficient of restitution at leisure by measurements of the plates. The paper presents the first instalment of the results of the work in the form of graphs (Figs. 1 to 5) showing the relation between the coefficient of restitution and the velocity of impact for polished spheres

[1] Pochettino, N. Cimento, Aug, 1914.

[2] Hodgkinson, Report Britt. Assoc., 1834.

[3] Vincent, Camb. Phil. Soc., Proc., August, 1900.

of equal radius of (1) brass, (2) aluminium, (3) hard bronze, (4) white marble and (5) lead.

2. EXPERIMENTAL METHODS AND RESULTS.

The impinging spheres were hung by bifilar suspensions, and were held apart and dropped electro-magnetically by the aid of small iron washers

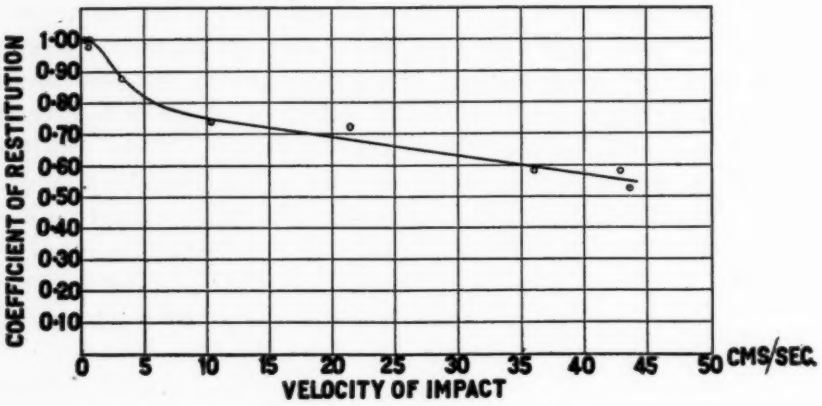

Fig. 1.

Brass spheres.

firmly fixed near the point of suspension. The motion of the balls before and after impact was recorded photographically in the following manner. A fine horizontal slit was illuminated by an electric arc, and an image of

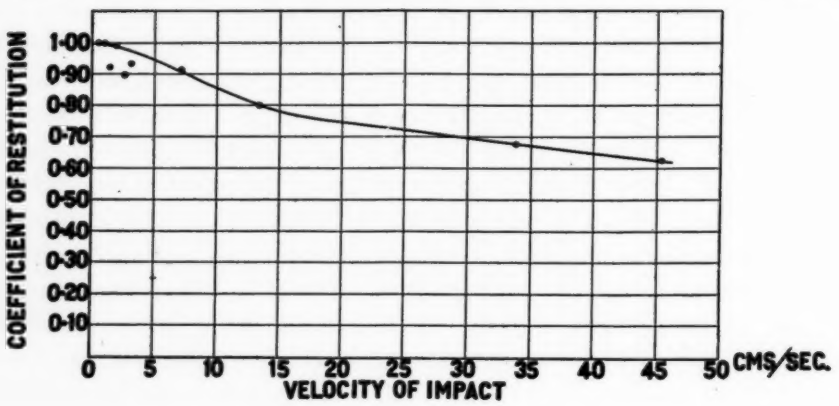

Fig. 2.

Aluminum Spheres.

it was focused on the lowest point of the circle on which the centers of the balls move when dropped, and so adjusted that when the balls hang

in the position of equilibrium, the image of the slit is just completely obscured, and appears only when the balls are drawn apart. A second lens refocuses the image of the slit seen between the balls, on a photographic plate which is caused to move vertically downwards when the balls are released. A fairly uniform vertical motion of the plate was obtained by

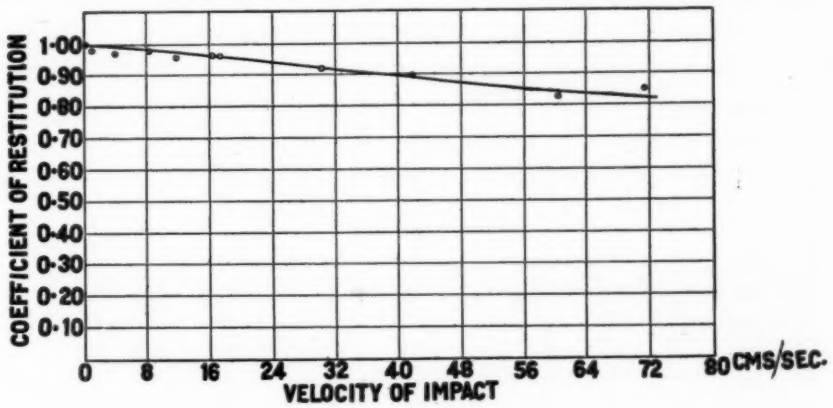

Fig. 3.

Hard Bronze Spheres.

suspending the dark slide from a string wrapped round the axle of a small fly wheel, and setting the latter in rotation. The dark slide moves down in vertical grooves at the back of the camera which is completely covered

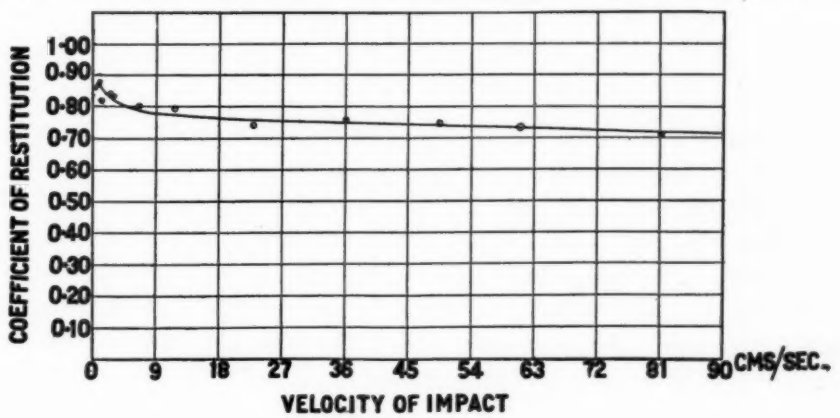

Fig. 4.

Marble spheres.

up with the exception of a small slit through which the light falls on the plate. With this arrangement the photograph can be taken without any need for specially darkening the room. The vibration of a tuning

fork of frequency 60 d.v. is simultaneously recorded on the photographic plate, and serves both as a check on the approximate uniformity of the motion and also as a scale of time against which the relative velocity of the balls is measured. A specimen photograph is reproduced as Fig. 6, see accompanying plate. The width of the wedges of light that appear immediately before and after the impact is measured at two or three points on each side, and gives directly the relative motion of the balls during the corresponding intervals of time. The ratio gives the coefficient of restitution. The magnification of linear dimensions on the plate is measured once for all by photographing a scale held in the plane containing the centers of the impinging spheres. By using long suspensions (2,

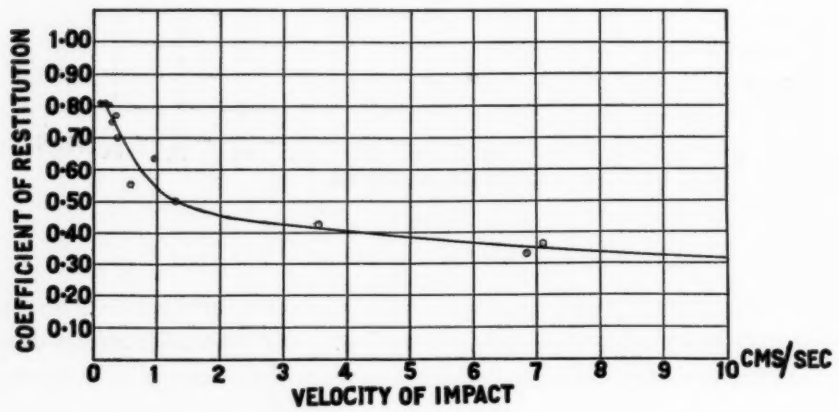

Fig. 5.

Lead spheres.

and sometimes 5 meters length), very small velocities can be obtained without an undue diminution in the amplitude of motion of the balls.[1]

The method was found very simple and convenient in practice, as the record once secured could be studied and measured at leisure. The most rapid way of measuring the photographs was to project them on a screen with a lantern, and to read off the widths of the wedges with an ordinary millimeter scale. An alternative which, however, proved much less convenient was to measure the photographs on a cross-slide micrometer.

It was found impossible to obtain regular and consistent results at low velocities of impact unless the surfaces of the balls were clean and highly polished. The choice of materials for the work was somewhat restricted

[1] In the course of the work, the curious observation was made that the suspensions of the spheres were set in oscillation as the result of impact in a mode which is initially the same as that of a bowed string observed by Helmholtz. (See the paper on Discontinuous Wave-motion by Raman and Appasamaiyar, Phil. Mag., January, 1916.)

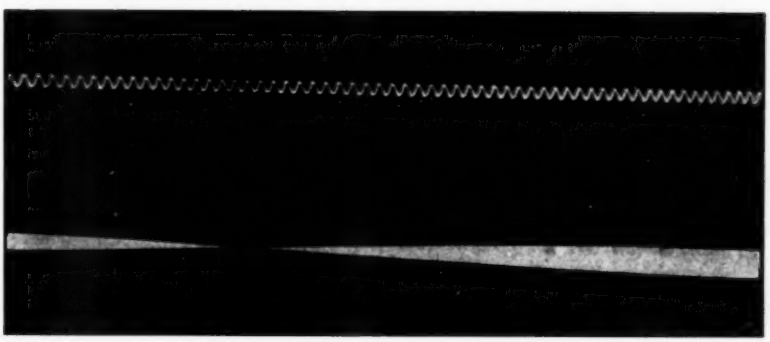

FIG. 6.

C. V. RAMAN.

by this consideration. According to recent views, polishing results in the formation of a surface film whose elastic properties are somewhat different from those of the bulk of the substance, and the complication introduced by this fact though undesirable, could not be avoided. Nevertheless the results indicated in Figs. 1 to 5 are sufficient to show that the coefficient of restitution makes a decided approach towa ds unity for a very small velocity of impact. The most noteworthy case is that of lead, in which this rise is particularly rapid. With marble spheres, it was found difficult to obtain a thoroughly satisfactory polish to begin with, and the breaking up of the crystalline structure at the place of contact as the result of impact was very evident to inspection. It was not therefore surprising that in the case of marble somewhat irregular results were obtained when it was attempted to push the observations down to very low velocities of impact. It is hoped when a suitable opportunity arises to continue this investigation in various directions, particularly as regards (*a*) the results obtained when the impinging spheres are of unequal diameters or of different materials; (*b*) to determine in what manner the apparent coefficient of restitution is affected by the density of the surrounding fluid, and (*c*) to give a fuller theoretical discussion of the results.

210, Bowbazaar Street,
Calcutta, India,
May 14, 1918.

GASES AND VAPORS FROM GLASS.

BY R. G. SHERWOOD.

THE mercury diffusion or condensation pump has made possible the realization of high vacua and, as a direct outcome, many new and interesting problems in research are encountered. A source of much trouble and error in a great deal of low pressure research arises from gases and vapors, which are liberated from the inner walls of the evacuated system. When such a system is being exhausted it is therefore common practice to simultaneously subject it for some time to a heat treatment at temperatures ranging between 250° and 450° Centigrade. This heating supposedly accelerates the liberation of occluded products, thereby making it possible to obtain and to maintain a good

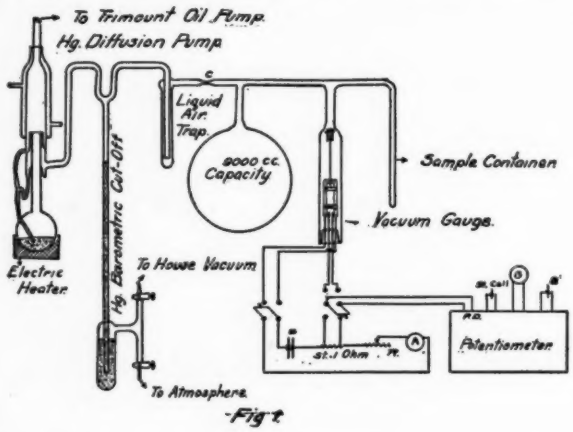

Fig. 1.

vacuum. The writer, who has been making a somewhat extended study of high vacua production, proposes to describe one of the methods used and to give a few of the results he has obtained on glass, at present the only material used in the construction of high vacuum systems.

APPARATUS.

The apparatus which is shown diagrammatically in Fig. 1 is constructed entirely of Corning G–702–P glass. This is a high melting point glass possessing a very small expansion coefficient and therefore admirably adapted for the construction of extensive vacuum systems.

A mercury diffusion pump, supported by a Trimount oil pump constitutes the pumping unit. A Kundsen vacuum gauge serves for the measurement of pressures within the system. A perspective drawing of this gauge without its envelope is shown in Fig. 2. The battery and poten-

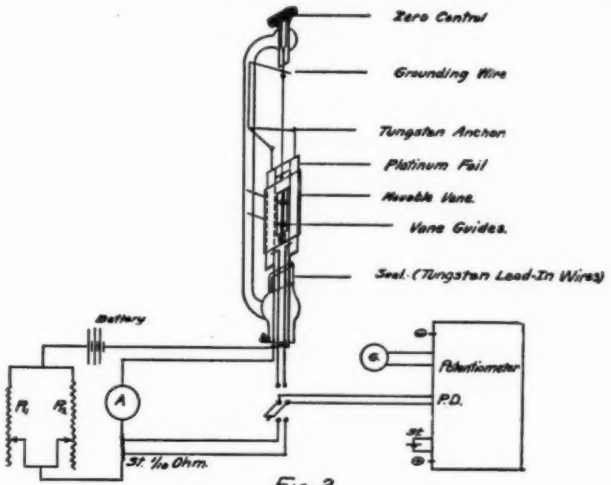

Fig. 2

tiometer measuring circuits are also shown in proper relation to the gauge. A detailed description of this type of manometer and of its method of operation has already appeared in the literature[1] and hence a few comments only will be made here relative to its use. For a pressure range between 10^{-4} and 10^{-8} mm. Hg this gauge makes a very sensitive and highly effective instrument, since the formula governing its use is valid at these low pressures. For pressure above 10^{-4} mm. Hg, however, the formula as derived by M. Kundsen becomes only approximate, except where the instrument is capable of very delicate adjustment wherein the suspended vane and the heated platinum foil can be kept in very close proximity to each other. For a general laboratory form of instrument, such as is shown in Fig. 2, such adjustment is not possible and hence a correction factor to Kundsen's Formula is necessary for the higher pressures, if it is to be used at all. Such a factor has been derived by the writer and has been found to hold quite satisfactorily in the case of dry air over the pressure range 10^{-4} to .05 mm. Hg.[2] For most work however the use of a calibration curve, made against a McLeod gauge, is to be preferred since it greatly facilitates the work and is fully as satisfactory where pressure differences are of fundamental importance, rather

[1] Kundsen, Aun'd Phy., IV., 32, 809, 1910; 44, 525, 1914. Woodrow, PHYS. REV., IV., 6, 491, 1914. Shrader and Sherwood, PHYS. REV., XII., 1, 70, 1918.

[2] Sherwood, PHYS. REV. (abstract), XI., 3, p. 241, 1918.

than absolute values. Such a curve, using a constant current through the gauge, has been used a great deal by the writer.

In order to keep the pressures within the best working range of the gauge and also to permit the use of rather good-sized specimens, it is advisable to have a system of considerable capacity. For these reasons the capacity of the system shown in Fig. 1 was chosen at about 9,000 c.c.

The constriction c serves a three-fold purpose: (1) It cuts down the speed of exhaustion so that data for a pressure versus time curve can be taken while exhaustion is in progress. (2) It maintains a constancy in the speed of exhaustion, independent of the speed of the pump providing the latter speed is sufficiently high. This is true only where the pressures within the system are low enough so that the mean free path of the molecules is comparable with or larger than the diameter of the communicating tubing or constriction. The constriction found suitable for the work described here had a bore of approximately one millimeter and a length of three millimeters; this permitted a speed of exhaustion of about 40 c.c. per sec. The speed of the pump was probably as high as 3,000 or 4,000 c.c. per sec. (3) In some instances it is desirable or even necessary to seal off the forward part of the system completely and the constriction affords a ready means for so doing.

METHOD.

For most of the results appearing in this paper a dynamic method was employed. The method is simple and for certain kinds of data has a distinct advantage over a static method. A preliminary procedure consists in pumping down the system to some low pressure, say 10^{-4} or 10^{-5} mm. Hg. Liquid air is also placed on the trap shown in Fig. 1 to remove mercury vapor. Because of the extreme slowness of exhaustion, due to the constriction and also to the continuous liberation of gases and vapors from the inner walls, all parts, with the exception of the sample container, are usually given a thorough torching out with a Bunsen flame, while exhaustion is in progress. When a sufficiently low pressure has been reached, that is a pressure say 10^{-4} or 10^{-5} mm. Hg, heat is then applied at the desired temperature to the sample container. The subsequent liberation of gases and vapors from the sample and container causes of course a rise in pressure within the system. If the change in the pressure with time is recorded and the corresponding curve plotted the total quantity of emission products liberated from the sample during the above time interval can be obtained. Such a pressure-time curve is shown in Fig. 3.

Let $P = f(t)$ be the functional relation between the pressure within the system and the time as represented by the above curve.

Let Q represent the total quantity of gas which has passed through the constriction at the end of the time t, measured in c.c. at unit pressure. If S be the speed of exhaustion in c.c. per second,—which will be con-

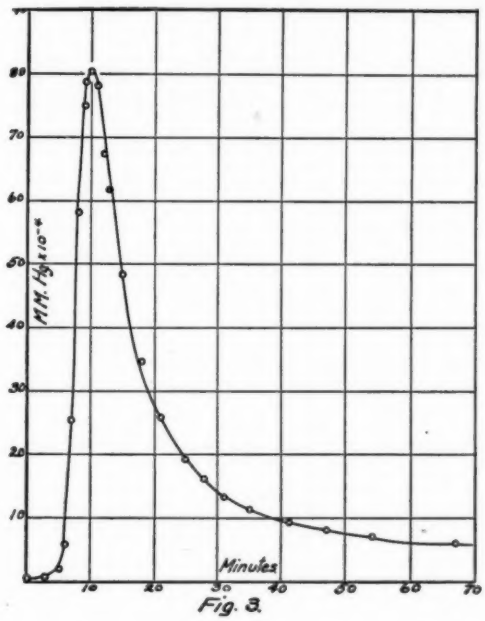

Fig. 3.

sidered constant over all pressures attained during the time interval in question, then it follows that

$$\frac{dQ}{dt} = PS = f(t) \cdot S \tag{1}$$

and

$$Q = S \int_0^t f(t)dt.$$

But from the theorem of mean value we have

$$\int_0^t f(t)dt = \overline{P}t = A$$

where $\overline{P}$ is the mean value for the pressure over the above time interval, A is the area between the curve, time axis and the ordinates P_0 and P_t. It follows then that

$$Q = SA. \tag{2}$$

When the speed S is known and the value of A is expressed in the proper units, the total quantity of gas which has passed through the constriction during the time t is at once obtained by means of equation (2).

In the special case where there is no liberation of gases or vapors into the system during exhaustion, the equation for the pressure time curve is readily obtained. If Q' represents the quantity of gas within the system at any instant during exhaustion and V the volume of the system then

$$Q' = PV. \tag{3}$$

Also the rate of flow through the constriction becomes,

$$\frac{-dQ'}{dt} = SP.$$

From relation (3) it follows that

$$\frac{-dP}{dt} = \frac{SP}{V}$$

or upon integration

$$\log P + \frac{St}{V} - \log P_0 = 0. \tag{4}$$

Where P_0 is the pressure within the system at the time $t = 0$. Hence the speed of exhaustion is:

$$S = \frac{(\log P_0 - \log P)V}{t}. \tag{5}$$

For all determinations of S by means of equation (5) it is necessary first to free the walls of the system from all occluded vapors and then to admit only dry air. The required data for determination of S can be obtained from the pressure-time curve taken during the reëxhaustion of the system. If S is constant a linear relationship between $\log P$ and t of course should follow.

The value of the speed S for the system and constriction above described was obtained from the mean of several determinations all of which showed that good constancy in the value of S could be depended upon for pressures as high as .02 mm. Hg.

The method above outlined for measuring the quantity of emission products from any given sample has the distinct advantage of being independent of adsorption effects. This follows from the fact that an equilibrium condition continually exists between adsorption, reëvaporation and exhaustion; whatever is adsorped by the walls of the system during a rise in pressure must accordingly be returned again to the system when the pressure falls. Delicate equilibria at very low pressures in this way make it possible to measure exceedingly small quantities of emission products even with the use of a less sensitive form of Kundsen gauge.

Results.

Although many data have been taken in the course of the work, but a few of the more significant results, capable of more or less definite interpretation, will be given here. The samples of glass used in the experiment consisted of the soft lead and soda glasses and also in particular Cornings (G–702–P) glass. From the results to be presented the writer proposes to discuss briefly two distinct kinds of gaseous evolution products obtained when glass is subjected to a heat treatment, the one solely the result of absorbed products while the other is in all probability due to a chemical decomposition of the various constituents of which the glass is composed.

Attention is directed to the curves shown in Fig. 4. Before the data

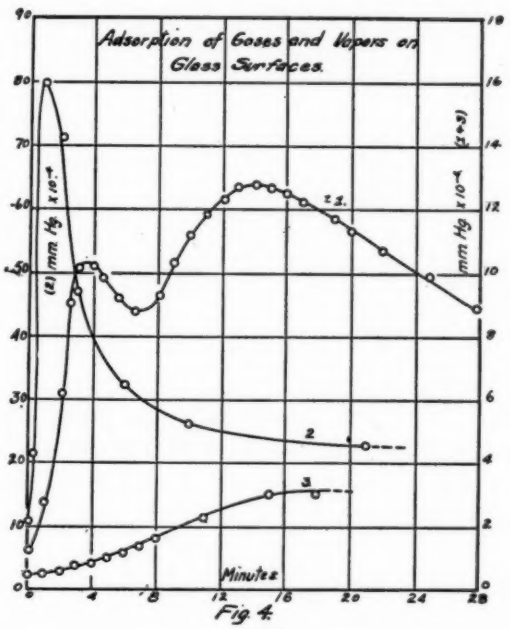

Fig. 4.

for curve (1) were taken the sample was first given a vacuum heat treatment at 200° C. for about one hour, the mercury cut-off shown in Fig. 1 meantime being operated to close off the pump. The vapors driven from the sample were of course all frozen out in the liquid air trap. When the heat was removed from the sample the liquid air was also removed from the trap, thus allowing the sample to again come into contact with the vapors and gases present. Two hours later liquid air was replaced on the trap, and the system again pumped down to a low vacuum. Heat was then applied at 350° C. with the result that all the adsorbed products were quickly removed. The small "hump" shown on curve 1 is explained

by the fact that the removal of the adsorbed products is accomplished before the characteristic effect for this temperature due to chemical reaction can take place; there is a small time interval therefore during which practically nothing is being removed from the sample, thereby resulting in a drop in the pressure as shown by the downward portion of the "hump." As soon however as the sample begins to attain the temperature of the furnace, the second kind of evolution becomes prominent and hence the further rise in the pressure shown by the curve. Curve 2, Fig. 4, shows the evolution resulting wholly from previously adsorbed gases and vapors. The downward portion of this curve is almost a pure exhaustion curve, which again shows how rapidly this kind of evolution takes place. Curve 3 shows the evolution of gaseous

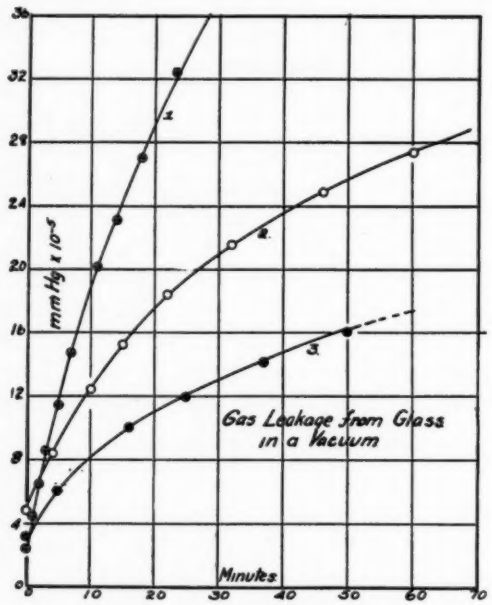

Fig. 5.

products which are not frozen out by the liquid air trap. In this case as well as in the two preceding cases the pump was cut off from the system, the liquid air trap functioning as a pump for the vapors only. This accounts for the fact that the curves all approach an asymptote above the initial value for the pressure.

Several experiments were made on the adsorbing properties of glass both for dry air and moist air. After thoroughly freeing the system from all occluded gases and vapors dry air was admitted and the system then pumped down again to a low pressure (10^{-4}–10^{-5} mm. Hg). The pump was then closed off and the rate of leak at room temperature recorded.

Three such leak curves are given in Fig. 5. Similar procedures using room air or air saturated with water vapor showed such a very slow leak that it was negligible in comparison with that obtained on dry air. The curves in Fig. 6 show the adsorption effects due to a mixture of oxygen and carbon dioxide which were allowed to enter the system dry and were

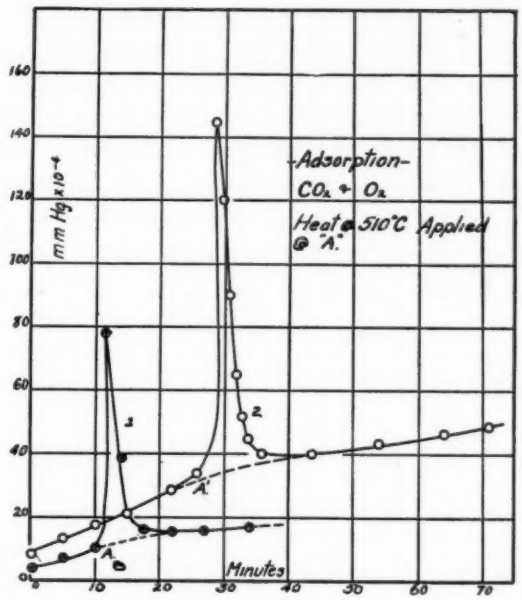

Fig. 6.

then pumped out again as in the above experiments. Heat was then applied to the sample container at the time indicated on the curves. The almost instantaneous evolution of the adsorbed CO_2 which is rapidly frozen out in the liquid air trap is well brought out by the curve. After the removal of the CO_2 the curves again become pure leakage curves due to the gradual leakage from the walls of the rest of the system. In still another experiment, after a pressure of about 10^{-4} mm. Hg, had been reached, the constriction was sealed off entirely and the subsequent change in pressure with time of standing at room temperature was observed. At the end of ten hours the pressure had risen to .0095 mm. Hg but did not increase materially after that. The application of liquid air to the sample container at the end of 20 hours caused but a very small lowering in the pressure, which shows conclusively that the above leak was due almost entirely to adsorbed air. Moreover, when heat at 500° C. was afterwards applied to the sample container, the further increase in pressure proved to be due almost wholly to vapors given off, since a second application of liquid air again reduced the pressure to practically its former value.

From the above results it is evident that there is a very strong adsorbing action between glass and water vapor or carbon dioxide and a rather weak adsorbing action toward air. The fact that dry air apparently leaks off from a dry glass surface very much more rapidly than does moist air is quite significant. The explanation suggested is that the active water molecules also have a very strong affinity for the gas molecules; consequently the air molecules are more strongly held to the surface in the pressure of water molecules than when the latter are not present. Moreover these adsorbed products are all very easily and quickly removed by the application of heat. Experiments using an entirely different method have substantiated the conclusions and have also showed that even a temperature treatment as low as 200° C. is practically as effective both as to time and degree as a higher temperature treatment when applied for the removal of adsorbed gaseous products.

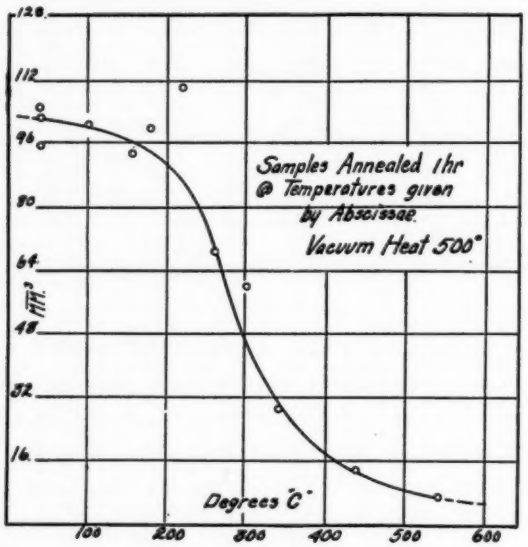

Fig. 7.

If there is an evolution of gaseous products from glass due to a chemical change taking place within the glass and which is greatly hastened under the influence of heat, it is quite logical to suppose that such a reaction should also occur whether the sample is heated within a vacuum or at atmospheric pressure. A series of experiments were therefore performed to determine if such were the case. Several samples, having the same dimensions and taken from the same batch of glass (G–702–P) were each annealed with its container at some chosen temperature for one hour, this temperature being different for each sample. After the samples

had been annealed in this way, each in turn was sealed onto the system and given a vacuum heat treatment at 500° C. The ordinates of the curve shown in Fig. 7 represent the total quantity of gaseous evolution products thus obtained from the particular sample whose preliminary annealing treatment is given by the corresponding abscissæ. Time of exposure to room air after the annealing of the sample proved to have little or no effect even after a period of several days had elapsed between the annealing and the sealing of the sample onto the system. If we assume that adsorption of gases and vapors upon a glass surface is confined to a layer approximately one molecule deep, then for the area of the glass heated, this quantity would correspond roughly to about 10 cubic millimeters. In this connection the above curve is of some interest, since it apparently approaches an asymptots corresponding to a value not greatly different from the above value. This apparently shows then that the gaseous products ordinarily removed from glass by heating are not adsorbed products entirely but that decomposition products

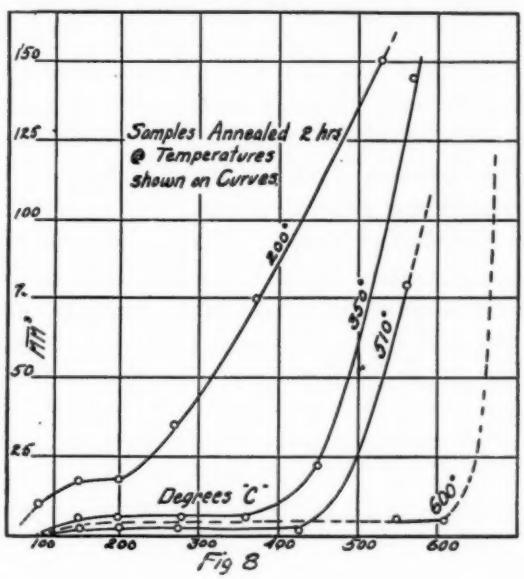

Fig 8

from the glass itself also result, the adsorbed products being only a very small part of the total. The curves in Fig. 8 give further information upon this same point. Each curve here shows the total emission from a particular sample which has previously been annealed for a two-hour period at the temperature designated upon the corresponding curve. The abscissæ represent the temperatures of the successive vacuum heat treatments applied to the sample. The ordinates give the integrated

total of the emission products up to and including the temperature shown by the corresponding abscissa.

From the above results it seems probable that, when glass is heated, new chemical equilibria must occur with the consequent evolution of gaseous reaction products. A second short heating at the same or at lower temperature produces no further, or at most but a relatively small effect. Referring again to Fig. 8 attention is directed to those portions of the curves just before and just after the annealing temperature had been reached in the treatment. The quantity represented by the flat portions of these curves is of a magnitude which could be accounted for by assuming an adsorbed layer of gas on the glass surface about one molecule deep or less. This quantity as the curves show and which has previously been brought out in the paper, comes off more or less completely at the lower temperatures.

SUMMARY.

1. A dynamic method has been described whereby the quantity of gas and vapor driven off from glass under the influence of heat can be measured.

2. There is a slow but persistent leakage of occluded gases from glass exposed to a vacuum even at room temperature and this leakage is more rapid for the inert gases than for vapors.

3. The adsorbed gaseous layer has been found to be of a thickness comparable to a layer one molecule deep.

4. There are two distinct kinds of gaseous evolution products from glass which is being heated; the one associated with adsorption, readily removable at temperatures as low as 200° C., and the other in all probability resulting from the formation of new chemical equilibria.

WESTINGHOUSE RESEARCH LABORATORY,
May 31, 1918.

THE ELECTROMAGNETIC VECTORS.

By H. Bateman.

§ 1. An electromagnetic field in the ether is usually specified by the values at each point and at each instant of two vectors E and H, but a more general specification is obtained by using the two vectors

$$F = E + \frac{1}{c}(v \times H),$$
$$Q = H - \frac{1}{c}(v \times E), \tag{1}$$

where v is an arbitrary velocity representing at each point and at each instant the velocity of an imaginary recording instrument and c is the velocity of light.

These vectors are of fundamental importance in electromagnetic theory for F is usually supposed to represent the force which the field would exert on a minute unit electric charge moving with velocity v and Q may be suppcsed to represent the force which the field would exert on a minute unit magnetic charge if such a thing could exist and move with velocity v.

On account of the importance of these vectors F and Q it will be worth while to get a clear conception of the way in which they vary when the field remains constant and v varies.

Let lines OE, OH, be drawn to represent the instantaneous values of E and H at any point in magnitude and direction and let circles of radii H and E and center O be drawn in planes at right angles to OH and OE respectively. If v is less than c the vector F is represented in magnitude and direction by the line FE where F is some point within the first circle while Q is represented by QH where Q is some point within the second circle. When v is greater than c the same construction may be used but the points F and Q may now lie outside their respective circles.

It is clear from this construction that F is a minimum when F is parallel to H and that Q is a minimum when Q is parallel to E.

When E and H are perpendicular and E is greater than H the point E lies in the plane of the first circle and outside the circle, consequently when v is less than c the direction of F lies within a certain angle bounded

by two straight lines in the directions of s_1 and $-s_2$ where s_1 and s_2 are the two real unit vectors s which satisfy the relations[1]

$$E + (s \times H) = ks, \qquad E \cdot s = k. \tag{2}$$

The vector Q can under the same conditions take any direction in a plane perpendicular to E.

In the case of the field of a moving electric pole one of the two vectors s is in the direction of the radius from the effective position of the pole, that is the point from which a disturbance travelling with velocity c must start in order to reach the point of observation O at time t.

When the electric pole moves with uniform velocity along a straight line the second vector s is in the direction of the radius to that position of the pole which can be reached by a disturbance starting from O at time t.

In the general case when E and H are not necessarily perpendicular, the directions of F and Q for $v < c$ are confined to certain quadric cones. If E is greater than H the angle of Q's cone is greater than that of F's. These two cones have the same focal lines which are in the direction of the two real unit vectors[2] s which satisfy the equations (2).

Relative to one of these vectors s the field vectors E and H may be resolved into longitudinal components $E \cdot s$ and $H \cdot s$ parallel to s and transverse components represented by the vectors $E - (E \cdot s)s$ and $H - (H \cdot s)s$ perpendicular to s. These latter components are perpendicular to one another and equal in magnitude as may be seen from the relations

$$(E \cdot H) = (E \cdot s)(H \cdot s) \tag{3}$$

$$E^2 - (E \cdot s)^2 = H^2 - (H \cdot s)^2 \tag{4}$$

which are easily derived from (2).

We may also divide up the energy in the field as follows:

Longitudinal electric energy $\frac{1}{2}(E \cdot s)^2$,
Longitudinal magnetic energy $\frac{1}{2}(H \cdot s)^2$,
Transverse electric energy $\frac{1}{2}E^2 - \frac{1}{2}(E \cdot s)^2$,
Transverse magnetic energy $\frac{1}{2}H^2 - \frac{1}{2}(H \cdot s)^2$.

It is clear from equation (4) that the transverse electric energy is equal to the transverse magnetic energy and that the Lagrangian function $\frac{1}{2}(E^2 - H^2)$ may be supposed to arise entirely from the longitudinal part

[1] They also satisfy the relations embodied in the vector equation

$$H - (s \times E) = s(s \cdot H).$$

[2] These vectors have been introduced for different purposes on previous occasions. Proc. London Math. Soc., Ser. 2, Vol. 8, 1910, p. 469; Vol. 10, 1911, pp. 7, 96; Mess. of Math., Vol. 14, 1915, p. 112.

of the field. It should be noticed that the longitudinal part of the field is *conjugate*[1] to the transverse part; in other words the invariants (for transformations of the theory of relativity) $E^2 - H^2$ and $(E \cdot H)$ for the total field are the sums of the corresponding invariants for the longitudinal and transverse parts. It should also be noticed that the energy in the total field is the sum of the energies of the longitudinal and transverse parts, consequently all the principal characteristics of the two component fields are additive.

The transverse field evidently has the characteristics of a pure radiant field or self conjugate field and in the case of an electric pole which at one instant has no velocity but a finite acceleration the transverse field relative to the vector s, which is in the direction of the radius vector from the pole, is the so-called wave of acceleration while the longitudinal field is the electrostatic field. When the electric pole has a finite velocity this description fails for the longitudinal field is the electric field represented by the radial component of the electric vector, while part of the transverse field depends on the velocity.

It is interesting to notice that the resolution of the energy into longitudinal and transverse energies is the same as far as the magnitudes are concerned whichever vector of type s is used. It should also be noticed that the total longitudinal energy

$$\tfrac{1}{2}(E \cdot s)^2 + \tfrac{1}{2}(H \cdot s)^2 = \tfrac{1}{2}[(E^2 - H^2)^2 + 4(E \cdot H)^2]^{1/2} \qquad (5)$$

is an invariant and is proportional in fact to Cunningham's principal stress. The longitudinal field disappears entirely when

$$E^2 - H^2 = 0 \qquad \text{and} \qquad (E \cdot H) = 0, \qquad (6)$$

that is, in the case of a pure radiant field or self-conjugate field. The two vectors of type s then coincide in direction with Poynting's vector.

It should be noticed that $v = cs$ is a special velocity for which both F and Q are parallel to v. The different possible directions of a velocity v such that F is parallel to v are of some interest and may be found as follows:

When F is parallel to v the vector $v \times E$ is perpendicular to F and is therefore perpendicular to $v \times H$. This means that the direction of v lies on a quadric cone which is the locus of a line OL for which the planes EOL and HOL are perpendicular. This cone has the lines OE, OH in one of its principal planes and has its circular sections perpendicular to them. It is clear that this cone is also the locus of directions for which Q can be parallel to v. If v and w have the same direction and v is a

[1] For the definition of conjugate fields see Bull. Amer. Math. Soc., Vol. 21, March, 1915, p. 299.

velocity for which F is parallel to v while w is a velocity for which Q is parallel to w, v and w are connected by the relation $vw = c^2$, consequently they cannot both be less than c and they cannot be equal unless they are both equal to c.

As OL moves round the cone the velocity v takes all values between O and ∞ twice over and so for any velocity there are generally two directions for which either F or Q is in the direction of v. Two generators of this cone are, of course, in the direction of the two real unit vectors s which satisfy equations (2). The directions of v for which F is perpendicular to v evidently lie in a plane perpendicular to E, similarly the directions of v for which Q is perpendicular to v lie in a plane perpendicular to H.

The directions of v for which F and Q are in the same direction are of some interest as these velocities v are possible velocities of the ether in the theory of E. Cunningham.[1] The directions are confined to two planes, one of which contains the two real unit vectors of type s and Poynting's vector while the other contains the two imaginary vectors of type s and Poynting's vector. These planes are in fact the two real planes through the real and imaginary focal lines of the two cones[2] already mentioned which limit the directions of F and Q respectively for $v < c$. As we have remarked elsewhere,[3] the extremity of a line OV representing the vector v lies on one of two straight lines which are polar lines with regard to a sphere center O and radius c, these lines intersect at right angles the line through O in the direction of Poynting's vector.

The lines FE representing the corresponding vectors F are such that F lies on one of two straight lines parallel to Poynting's vector. These lines may be obtained by drawing through E two planes parallel to the two planes previously mentioned and finding where these planes meet the plane through O perpendicular to H. There is a similar construction for the corresponding vectors Q.

When Q is perpendicular to F we have an equation

$$(c^2 - v^2)(E \cdot H) = -(v \cdot E)(v \cdot H),$$

for v which indicates that the extremity of a line OV representing this velocity lies on a quadric surface having the cone swept out by OL as asymptotic cone. The region for which v is less than c is bounded by the two planes through O perpendicular to E and H and consequently by the two diametral circular sections of the quadric. If we are given the direction of F the corresponding value of v for which Q is perpendicular

[1] For this remark see S. B. McLaren, Phil. Mag., 26, 1913, p. 636.

[2] These cones are supposed to have their vertices at O.

[3] Phil. Mag., 34, 1917, p. 405; Mass. of Math., 14, 1915, p. 110.

to F is uniquely determined except in the case when OV is in the plane containing OH and the line through O in the direction of Poynting's vector. The correspondence between the points F and Q when F is perpendicular to Q is a one to one quadratic transformation. The point at infinity in the direction of Poynting's vector and the points in which the plane OEH meets the circle are the singular points in each plane. This completes the description of the geometrical properties of an electromagnetic field which is supposed to be explored by a crowd of imaginary observers moving in an arbitrary manner.

We have been acting on the assumption that F and Q are the quantities that are observed directly and that E and H are secondary quantities which may be derived from them. The quantities E and H are usually regarded as the primary quantities on account of the simplicity of the field equations which they satisfy but it should be noticed that the electromagnetic laws can also be expressed directly in terms of F and Q. This is a known result but it may be worth while to recall it at this point.

§ 2. In the year 1908 two very important papers on electromagnetic theory were published. One of these was Minkowski's paper on the electrodynamical equations for moving bodies,[1] a paper which soon influenced mathematical thought very considerably and received world wide attention. The other paper was by Mr. Richard Hargreaves, of Southport, England, and was entitled "Integral forms and their connection with physical equations." This paper which is perhaps the more important of the two, contains two new presentations of the principles of electromagnetism in terms of space-time integrals. This at once places the time coördinate on the same level as the other coördinates and suggests the idea of space-time vectors just as in Minkowski's work. The chief importance of Mr. Hargreaves' work lies, however, in the fact that it throws light at once upon the nature of the solutions of the electromagnetic equations and that the principles are presented in a form which is independent of the choice of the space and time coördinates. The last circumstance enables one to obtain the transformations of the theory of relativity in a simple and natural manner and makes it easy to obtain the invariants by a simple application of the methods of the absolute calculus of Ricci and Levi Civita.[3] The first two theorems which are usually written in the form

[1] H. Minkowski, Gött. Nachr., 1908.

[2] R. Hargreaves, Cambr. Phil. Trans., Vol. 21, 1908, p. 107. Some interesting developments and applications of Hargreaves' theorems have been made in an enthusiastic way by M. de Donder in Belgium, Bull. de l'Acad. roy. de Belgique (Classe des Sciences), 1909, p. 66; 1911, p. 3; 1912, p. 3.

[3] H. Bateman, Proc. London Math. Soc., Vol. 8, 1910, p. 223.

$$\int\int [E_x d(y, z) + E_y d(z, x) + E_z d(x, y) - cH_x d(x, t) - cH_y d(y, t) - cH_z d(z, t)]$$
$$= \int\int\int [\rho d(x, y, z) - \rho w_x d(y, z, t) - \rho w_y d(z, x, t) - \rho w_z d(x, y, t)]$$
$$\int\int [H_x d(y, z) + H_y d(z, x) + H_z d(x, y) + cE_x d(x, t) + cE_y d(y, t) + cE_z d(z, t)] = 0$$

indicate the invariance of the electromagnetic equations under conditions in which the integrands in the integrals are invariants. Now the first integral certainly vanishes when the moving surface of integration is made up of moving lines of electric force[1] and this indicates the invariance of moving lines of electric force under the transformations of the theory of relativity. This theorem is true for all transformations of coördinates when the ideas of general relativity are adopted and the vectors in the second of the two equations are not assumed to be the same as in the first.

A moving line of electric force thus assumes the form of a definite physical entity when we adopt the view that invariants are symbols for physical entities which are independent of the measuring apparatus or method of observation.

We have already shown that Mr. Hargreaves' theorems can be presented in a form in which ordinary surface and volume integrals are used[2] and as this gives the desired expression of the electromagnetic laws in terms of the vectors F and Q the results will be quoted here in vector notation.

Let us assume that throughout a certain region of space a time is associated with each point in space by means of a relation of type $t = f(x, y, z)$, where f is a uniform function. This time may be supposed to be the time at which an observation is made at the point in question. We shall suppose, moreover, that the velocity of the observer or observing instrument is determined by the equation

$$v = c^2 \Delta f, \tag{7}$$

then with the notation of § 1, we have the two theorems

$$\int F_n d\sigma = \int \rho \left[1 - \frac{1}{c^2}(v \cdot w) \right] d\tau, \tag{8}$$

$$\int Q_n d\sigma = 0, \tag{9}$$

where $d\sigma$ denotes an element of surface, $d\tau$ an element of volume, the suffix n denotes that the normal component of the vector is taken, ρ

[1] Phil. Mag., Vol. 34, 1914, p. 405.

[2] See last reference but one.

denotes the volume density of electricity and ρw the convection current. In these theorems the surface integrals are supposed to be taken over a closed surface and the volume integral over the volume enclosed by this surface. In the ether of course the volume integral disappears and we have

$$\int F_n d\sigma = \int Q_n d\sigma = 0.$$

The equation (8) may be regarded as a generalization of Gauss' theorem and may be assumed to hold whether the behavior of the quantities involved permits an application of the ordinary form of Green's theorem or not.[1] Calling $F_n d\sigma$ the flux across the element of surface $d\sigma$ the theorem may be interpreted to mean that the flux across a closed surface is equal to the charge inside. This, of course, is in a generalized sense, for it must be remembered that quantities are measured at different points of space at different times. Since the flux across a surface made up of moving lines of electric force is zero we may conclude that the flux across a cross section of a tube made up of moving lines of electric force is constant along the tube provided there is no electricity within the tube between the two sections under consideration at the times specified by the law of observation $t = f(x, y, z)$. We must generally assume that observations are made throughout a region of space by ∞^3 observers. This is necessary for instance if the observations are to be simultaneous but in some cases it is convenient to assume that sets of observations are made by ∞^2 observers travelling along specified paths. In particular, if the observers are supposed to travel along straight lines with the velocity of light the velocity v may represent the velocity of the observer and the time $t = f(x, y, z)$ the time at which he reaches the point (x, y, z).

Let us take the case in which v is at each point in the direction of one of the unit vectors s in an electromagnetic field and that sets of observations are made by imaginary observers who travel along straight lines with the velocity of light. The vectors F and Q then represent the longitudinal part of the field and the electromagnetic laws give us the properties of this part of the field alone. Thus if the field is that of a moving electric pole and the observers are supposed to start from a particular position of this pole and travel away from it with the velocity of light our theorems enable us to study the properties of the field which is left when the transverse wave is subtracted from the total field. In other words our observers do not record the effects of the transverse wave because they travel with it at the speed of light.

[1] In this way we may avoid the difficulty that occurs on p. 21 of my book "Electrical and Optical Wave Motion." The above theorems are more general than the electromagnetic equations.

Mr. Hargreaves' second set of theorems indicate the way in which space time integrals change when a small variation is made of the region of integration; they give for instance the rate of change of the flux across a closed surface when the surface is in motion.[1] Let us suppose that the change of times and region of integration are made by moving the observer at each point (x, y, z) to some consecutive point $(x + \delta x, y + \delta y, z + \delta z)$ at a velocity u which is a function of x, y, z, t and let the new time of observation be the time at which each observer arrives at his new position.

We shall quote Hargreaves' theorem for the case of a volume integral which in our notation is written in the form

$$\int \rho \left[1 - \frac{1}{c^2} (v \cdot w) \right] d\tau.$$

ρ and w are supposed here to satisfy the equation of continuity

$$\frac{\partial \rho}{\partial t} + \operatorname{div} \rho w = 0,$$

so that the integral may be regarded as representing the total amount of electricity within a closed moving surface when the observations are made at times specified by the law $t = f(x, y, z)$, and w represents the velocity of the electricity, ρ its density. The rate of change of this volume integral is now represented by an integral of type

$$\int F_n' d\sigma$$

over the boundary of the closed surface where

$$F' = E' + \frac{1}{c} (v \times H')$$

and E' and H' are defined by the equations

$$E' = \rho(u - w), \qquad H' = \frac{1}{c} \rho[w \times u].$$

This result is of interest because it indicates that if we start with the idea of a fluid electricity moving according to the law of continuity we can, by a process analogous to differentiation, derive from it a field of two vectors E' and H' which are expressible directly in terms of the quantities ρ and ρw which specify the flow of electricity in the original field. The quantities ρ' and $\rho' w'$ which are connected with E' and H' by the equations

$$\operatorname{div} E' = \rho', \qquad c \operatorname{rot} H' = \frac{\partial E'}{\partial t} + \rho' w',$$

[1] This provides us with the generalized forms of two theorems given by H. A. Lorentz, Encyklopädie der Math. Wiss., Bd. V., § 13, 1903, p. 119 and J. Larmor, Proc. Int. Congr. of Math., Cambridge, 1912, Vol. 1.

are different from ρ and ρw because the total amount of electricity within a closed surface in the field E', H' represents the rate of change of the amount of electricity within this closed surface in the original field. Strictly we ought not to use the same word electricity in the two cases because the quantities ρ and ρ' are not of the same dimensions. To avoid confusion we may call ρ' density of electricity and ρ density of *proto-electricity*. It must be clearly understood that quantity of electricity represents the rate at which a quantity of proto-electricity appears to change in magnitude when successive observations are made by a crowd of imaginary observers moving in such a way that the velocity of the observer at (x, y, z, t) is $u(x, y, z, t)$. This vector u may of course be the same as the vector v but it is not necessary to assume that the velocity of an observer is the same as that of his instrument of observation.

It should be noticed that in our derived field E', H' we have

$$E' \cdot H' = 0 \qquad \text{and} \qquad E' > H'$$

where the inequality holds provided at least one of the velocties u, w is less than c. To prove this inequality let us draw lines OU, OW to represent the velocities u and w in magnitude and direction. The quantity E'^2 is then represented by the square of the distance UW while H'^2 is represented by the square of the distance UW multiplied by the square of the perpendicular from O on UW and divided by the square of c. Clearly then E' is greater than H' if the perpendicular is less than c and this is certainly true if one of the quantities u, w is less than c. If both u and w are greater than c it does not necessarily follow that E' is not greater than H' for the line UW may still cut a sphere of radius c in real points and so be at a distance from O less than c.

The idea that quantity of electricity represents a rate of change of a quantity of proto-electricity may perhaps account for the existence of two types of electricity, positive and negative, because even if a quantity of proto-electricity is always represented by a positive number, its rate of change may be either positive or negative.

This suggests that the plan of deriving electromagnetic fields from proto fields of moving proto-electricity by a process analogous to differentiation may be very useful. It should be remarked at the outset that the type of differentiation considered above is only a particular case of a more general type of differentiation in which two proto fields which differ slightly from one another are subtracted and a limiting process carried out after all the small quantities obtained are divided by the same small quantity.

Of course in a proto field we can consider two vector functions E and H which are connected with ρ and ρw by the electromagnetic laws and it seems likely that fields which satisfy the electromagnetic equations and do not possess the characteristics of the types of electromagnetic fields with which we are familiar may be really proto fields. The fields whose singularities consist of moving curves are a case in point for when two such fields are superposed so that the singular curves overlap it is possible to obtain a cancelling of singularities with the result that in the total field there are only point singularities. In other words line charges in the proto field may give rise to isolated electric charges in the derived field. In the original form of Sir Joseph Thomson's theory of moving Faraday tubes the analysis indicates that a Faraday tube always consists of the same particles of electricity.[1] This result may seem strange to most scientists but the explanation is that a field of Thomson's type is probably a proto field and that really a Faraday tube always consists of the same particles of *proto-electricity* while when an electromagnetic field is derived from this proto field by a process analogous to differentiation the electric charges appear only at the ends of the tube. The foregoing remarks indicate that the interpretation of the electric and magnetic vectors in terms of proto-electricity may be simpler than any interpretation in terms of the electricity of their own field and this we shall now endeavor to show. It may be remarked that the dynamical laws of motion for proto-electricity are unknown, it is possible that they are of the first order instead of the second and that the accelerations in the Newtonian laws arise in the process of differentiation by which an electromagnetic field is derived from a proto field.

§ 3. To begin with it will be convenient to assume that a proto-electric charge can be either positive or negative. The negative charges can be eliminated eventually, by adding positive constants to all the charges, but this may not be necessary.

Let us suppose that a positive charge e and a negative charge $-e$ separate at a point O and travel along straight lines with the velocity of light and let the points A and B respectively represent their positions at some time t. If T denotes the time which has elapsed from the moment of separation up to this instant, we have

$$OA = OB = cT.$$

Now consider two imaginary observers, one of whom travels with the point A while the other travels with the point B. Let the unit vectors s_1 and s_2 specify the directions of motion of A and B. If we imagine

[1] See H. Bateman, Phil. Mag., Vol. 34, 1917, p. 405.

a field of two vectors E and H to exist at the two moving points A and B and nowhere else, A may be supposed to record the vectors

$$F_1 = E + (s_1 \times H),$$
$$Q_1 = H - (s_1 \times E),$$

while B records the vectors

$$F_2 = E + (s_2 \times H),$$
$$Q_2 = H - (s_2 \times E).$$

Now a natural vector for A to record is $ecTs_1$ and a natural vector for B to record is $-ecTs_2$; we shall therefore consider the consequences of assuming that

$$\begin{aligned} E + (s_1 \times H) &= -ecTs_1, \quad & H - (s_1 \times E) &= 0, \\ E + (s_2 \times H) &= ecTs_2, \quad & H - (s_2 \times E) &= 0. \end{aligned} \tag{10}$$

These equations are quite consistent with one another and they indicate that s_1 and s_2 are the two vectors of type s defined by equations (2) for the two points where the field exists. Solving these equations we get

$$E = ecT\frac{s_2 - s_1}{1 - s_1 \cdot s_2}, \qquad H = ecT\frac{s_1 \times s_2}{1 - s_1 \cdot s_2}. \tag{11}$$

The electric vector is thus in the direction of the line AB but is not simply proportional to the length of this line for there is an additional factor depending on the angle between the vectors s_1 and s_2. The magnetic vector H is perpendicular to both these vectors and to E.

It should be noticed that

$$E^2 - H^2 = e^2c^2T^2, \qquad E \cdot s_1 = -ecT, \qquad E \cdot s_2 = ecT,$$

$$E^2 = e^2c^2T^2\frac{2}{1 - s_1 \cdot s_2}, \qquad H^2 = e^2c^2T^2\frac{1 + s_1 \cdot s_2}{1 - s_1 \cdot s_2},$$

$$E \times H = e^2c^2T^2\frac{s_1 + s_2}{1 - s_1 \cdot s_2}.$$

If the flow of energy is supposed to be indicated by Poynting's vector a puzzling result is obtained for since the field exists only at the two moving points A and B it is difficult to understand how energy can get away from these points, which it would do if the flow were in the direction of the vector $s_1 + s_2$. The probable explanation of this paradox is that the flow is modified owing to the presence of the charges and the work which is done on them by the field. We are interested in the behavior of the total energy and so the concealed energy which is equivalent to the work done must be taken into account and its influence on the total flow must also be considered.

We shall consequently take the total energy at A to be a multiple of[1]

$$\tfrac{1}{2}(E^2 + H^2) + \tfrac{1}{2}(E\cdot s_1)^2 = E^2$$

and the flow of energy to be represented by the same multiple of

$$c(E \times H) + cE(E\cdot s_1) = cE^2 s_1.$$

Similarly the total energy at B will be assumed to be a multiple of

$$\tfrac{1}{2}(E^2 + H^2) + \tfrac{1}{2}(E\cdot s_2)^2 = E^2$$

and the flow of energy the same multiple of

$$c(E \times H) + cE(E\cdot s_2) = cE^2 s_2.$$

This makes the flow of energy take place in the directions of motion of A and B which seems right. The lack of symmetry with respect to E and H arises on account of the fact that $(H\cdot s_1)$ and $(H\cdot s_2)$ are both zero.

Let us now use our expressions for E and H to build up an electromagnetic field in the ether. Instead of supposing that the separation of charges takes place at only one point at one instant we shall suppose that a separation is continually taking place in the neighborhood of a moving point S whose coördinates at time τ are $\xi(\tau)$, $\eta(\tau)$, $\zeta(\tau)$. We shall suppose that in the process of separation which takes place during the interval of time from τ to $\tau + d\tau$ concentrated charges of magnitudes $f'(\tau)d\tau$ and $-f'(\tau)d\tau$ are fired out in the directions specified by the unit vectors s_1 and s_2 respectively and that diffuse charges which will just balance the concentrated charges are shot out in all directions in such a way that each compensating diffuse charge is distributed uniformly throughout the shell bounded by the two spheres of radii $c(t-\tau)$ and $c(t-\tau-d\tau)$ whose centers are the positions of S at times τ and $\tau + d\tau$ respectively. The density of the charge compensating $f'(\tau)d\tau$ is easily seen to be

$$\frac{f'(\tau)}{4\pi r\nu},$$

where

$$\nu = \xi'(\tau)[x - \xi(\tau)] + \eta'(\tau)[y - \eta(\tau)] + \zeta'(\tau)[z - \zeta(\tau)] - c^2(t-\tau),$$

$$r = c(t-\tau).$$

On account of its displacement this charge provides us with the field

$$E_1 = \frac{f'(\tau)}{4\pi\nu}\,\frac{s - s_1}{1 - s\cdot s_1}, \qquad H_1 = \frac{f'(\tau)}{4\pi\nu}\,\frac{s_1 \times s}{1 - s\cdot s_1},$$

where rs denotes the vector whose components are $x - \xi(\tau)$, $y - \eta(\tau)$, $z - \zeta(\tau)$, respectively. It should be noticed that the *density* of the

[1] The term $\frac{1}{2}(E\cdot s_1)^2$ represents the potential energy gained on account of work done against the force.

diffuse charge is used here in place of e and so E_1, H_1 are in a sense densities of the corresponding quantities when e is used. The density of the diffuse charge compensating $-f'(\tau)d\tau$ is likewise

$$\frac{-f'(\tau)}{4\pi r v}$$

and this provides us with the field

$$E_2 = -\frac{f'(\tau)}{4\pi v}\frac{s - s_2}{1 - s\cdot s_2}, \qquad H_2 = -\frac{f'(\tau)}{4\pi v}\frac{s_2 \times s}{1 - s\cdot s_2}.$$

Superposing the two fields the densities of the two diffuse distributions of charge cancel out but we are left with an electromagnetic field specified by the vectors

$$\begin{aligned} E &= \frac{f'(\tau)}{4\pi v}\left[\frac{s - s_1}{1 - s\cdot s_1} - \frac{s - s_2}{1 - s\cdot s_2}\right], \\ H &= \frac{f'(\tau)}{4\pi v}\left[\frac{s_1 \times s}{1 - s\cdot s_1} - \frac{s_2 \times s}{1 - s\cdot s_2}\right]. \end{aligned} \tag{12}$$

In these expressions it must be remembered that τ is defined in terms of x, y, z and t by means of the equation

$$[x - \xi(\tau)]^2 + [y - \eta(\tau)]^2 + [z - \zeta(\tau)]^2 = c^2(t - \tau)^2, \qquad \tau \leq t.$$

If we introduce the auxiliary quantities

$$\alpha = \frac{f(\tau)}{4\pi}, \qquad \beta = \log\frac{1 - s\cdot s_1}{1 - s\cdot s_2},$$

it is easy to see that

$$E = \frac{1}{c}\left(\nabla\alpha\frac{\partial\beta}{\partial t} - \nabla\beta\frac{\partial\alpha}{\partial t}\right), \qquad H = \nabla\alpha \times \nabla\beta, \tag{13}$$

and that

$$\text{rot } H = \frac{1}{c}\frac{\partial E}{\partial t}, \qquad \text{div } E = 0,$$

$$\text{rot } E = -\frac{1}{c}\frac{\partial H}{\partial t}, \qquad \text{div } H = 0.$$

This means that our field vectors satisfy Maxwell's equations and that these equations are a consequence of our assumption that proto-electric charges travel along straight lines with the velocity of light and our additional hypothesis for the specification of the electric and magnetic vectors.[1] This hypothesis is that the field vectors at the two moving points A, B are the same at any instant, that the longitudinal components

[1] Another derivation of Maxwell's equations from elementary assumptions is given by Leigh Page, Am. Jour. Sci., 38, 1914, p. 169. He uses the theory of relativity and the assumption that each electric point charge is a center of uniformly diverging tubes of strain.

of the electric vector are $-ecT$ and $+ecT$ respectively and that the longitudinal components of the magnetic vector are zero. By longitudinal components we mean components parallel to s_1 and s_3. Another way of looking at the matter is to regard the coefficients of ecT in formulæ (11) as direction ratios of the moving line AB. Since each vector has three components there are six direction ratios.

The field specified by the formulæ (12) is a simple generalization of the one described by Heaviside[1] in 1901. If the function $f'(\tau)$ varies continuously the concentrated charges form two moving curves along which the above expressions for E and H are infinite but it is not certain that these expressions are valid for these moving curves. Since the field contains line charges it should, perhaps, be regarded as a proto field from which an ordinary electromagnetic field may be derived by a process analogous to differentiation. An appropriate process is described fully in a paper which will appear shortly in the Proceedings of the London Mathematical Society. In the simple case when the line charges are stationary except for motion along the line the process is simply a differentiation with regard to one coördinate z, for we may write

$$\alpha = t - \frac{r}{c}, \qquad \beta = \log\frac{r-z}{r+z}, \qquad r^2 = x^2 + y^2 + z^2.$$

The resulting field is the electrostatic field of an electric pole at the origin.

It should be noticed that a field of type (12) from which the field of a moving electric pole may be obtained by a process analogous to differentiation is a pure radiant field in which the vectors E and H satisfy the conditions

$$(E \cdot H) = 0, \qquad E^2 - H^2 = 0.$$

As this field is of an elementary nature the character of the energy ought to be determined and this is a matter which we shall now discuss.

§ 4. I think most authorities agree that the flow of energy in an ordinary electromagnetic field may be represented by means of Poynting's vector[2] $S = cE \times H$ and that the volume density of electromagnetic energy is $W = \frac{1}{2}(E^2 + H^2)$.

If no electricity is present so that no work is being done at the point under consideration the question arises as to whether the transfer of energy can be represented as a motion of all the energy in a single direction.

The fact that the equation of continuity

$$\frac{\partial W}{\partial t} + \operatorname{div} S = 0$$

[1] O. Heaviside, The Electrician, Nov. 29, 1901. Electromagnetic Theory, Vol. III., p. 122.

[2] For a recent discussion of the matter see a paper by G. H. Livens, Phil. Mag., 34, 1917, p. 385.

is satisfied suggests that it is sometimes possible to adopt this view, and as the velocity of motion

$$\frac{c}{W}\{E^2H^2 - (E\cdot H)^2\}^{1/2}$$

is never greater than c, the velocity of light, this way of looking at the matter seems at first quite reasonable. It is of some interest, however, to compare this velocity with Cunningham's velocity of the ether,[1] which we shall denote by the symbol u.

Let us consider the hypothesis that the energy in a field can be regarded as energy of motion of a single entity or a group of entities having practically the same motion when the velocity of flow is a possible velocity of the ether.

Taking first the simple case in which $E\cdot H = 0$, the velocity u is governed by one of the equations $H = u \times E$, $cE + u \times H = 0$ but the component of u perpendicular to S is indeterminate. If now u is in the direction of S, u is equal to either $c\cdot H/E$ or $c\cdot E/H$ and it is clear that u is equal to the velocity of flow of the energy only in the case when $E^2 = H^2$.

A similar result is obtained when we do not make the initial assumption $E\cdot H = 0$; for in this case the velocity u is governed by the equation[2]

$$ch = u \times e,$$

where

$$h = \lambda H + \mu E, \qquad e = \lambda E - \mu H,$$

and λ and μ are chosen so that $e\cdot h = 0$. Since $e \times h = (\lambda^2 + \mu^2)(E \times H)$ it appears that when u is in the direction of S, $u\cdot e = 0$ and we have

$$c^2h^2 = u^2e^2.$$

Now

$$e^2 + h^2 = (\lambda^2 + \mu^2)(E^2 + H^2)$$

and

$$e^2h^2 = (\lambda^2 + \mu^2)^2\{E^2H^2 - (E\cdot H)^2\},$$

hence u is equal to the velocity with which the energy flows only when $e^2 = h^2$ and then it follows that $E^2 = H^2$ and $E\cdot H = 0$.

Thus it is only in a simple radiant field for which the above equations are satisfied that the energy may be supposed to flow with the same speed as the ether. I have adopted the view elsewhere[3] that the energy in such a field is entirely kinetic energy or energy of motion but this view is rather unorthodox and may perhaps be questioned. The point at issue depends on the definition of the different kinds of energy and also

[1] Proc. Roy. Soc. London, 83, 1909, p. 110. The Principle of Relativity, Ch. XV.

[2] See H. Bateman, Mess. of Math., 14, 1915, p. 111.

[3] Proc. National Academy of Sciences, May, 1918.

on the question whether the usual identification of magnetic energy with kinetic energy is valid for all types of field. Perhaps it is advisable to distinguish between kinetic energy and energy of motion, as is done for instance by Page,[1] and to use the term kinetic energy only when energy can be expressed in the form $T = \frac{1}{2}mu^2$ where u is a velocity and m the transverse mass. According to this view the kinetic energy in the electromagnetic field of a moving electron can be represented by $\frac{1}{2}H^2$ per unit volume, this being the usual expression adopted by Larmor and others. This view seems to be strengthened by the form of the Lagrangian function in the principle of least action but it may not be right to adopt the expression $\frac{1}{2}H^2$ for the kinetic energy per unit volume in the ideal case of a magnetic particle for we may expect from symmetry that $\frac{1}{2}E^2$ is the correct expression. Since, moreover, a simple radiant field cannot generally be derived by superposition from the fields of electric poles moving with velocities less than c we cannot conclude from the above that the kinetic energy per unit volume is in this case represented by $\frac{1}{2}H^2$, although there may be some other way of arriving at this result. At any rate it does not seem right to assume that $\frac{1}{2}H^2$ per unit volume completely represents the *apparent energy of motion*[2] in the field, for it seems reasonable to adopt the view that there is no apparent energy of motion when the field is static and there is no flow of energy. Assuming that a field in the ether is static when there is no flow of energy a static field may be characterized by either $E = 0$ or $H = 0$, the general requirement being that E should be parallel to H. When a field is static it is possible for Cunningham's velocity u to be zero and conversely if u can be zero the field is static.

According to the above view there is no apparent energy of motion either in an electrostatic field or a static magnetic field but in both cases there is concealed energy of motion if we adopt the ideas of Thomson and Hertz. This concealed energy of motion may perhaps be brought into evidence by building up the static fields from simple radiant fields of the type for which $E^2 = H^2$ and $E \cdot H = 0$, for in each field of this type there is certainly energy of motion.

In what follows we shall adopt the hypothesis that the energy in a simple radiant field is entirely energy of motion. In justification of this we may, perhaps, reason as follows, but the argument is not very conclusive.

In the case when $cH = u \times E$ and u is parallel to S the kinetic energy

[1] Am. Jour. of Science, 40, Aug., 1915.

[2] This term is used because according to the views of Sir Joseph Thomson—The Applications of Dynamics to Physics and Chemistry, 1888, p. 15—all energy is ultimately kinetic energy or energy of motion.

$\frac{1}{2}H^2$ and the momentum $(1/c^2)S$ can be accounted for by assuming that a particle with transverse mass $m = E^2/c^2$ moves with velocity $u = c \cdot H/E$ in the direction of S. Now in the case of a moving electron the energy which it has acquired in virtue of its motion may be expressed in terms of the velocity u and transverse mass m by means of the formula[1]

$$mc^2[1 - \sqrt{1 - \beta^2}][1 + \tfrac{1}{4}\sqrt{1 - \beta^2}], \qquad (14)$$

where $\beta = u/c$. If this formula is supposed to be applicable in the case of a particle whose transverse mass remains finite when $u = c$ the expression for the energy of motion becomes mc^2 and this is equal to E^2 in the case of the electromagnetic field. The view that the energy is entirely energy of motion when $E^2 = H^2$ and $E \cdot H = 0$ is thus in accordance with the above expression. Of course if the transverse mass is finite when $u = c$ the particle must be supposed to have no energy when it is at rest.

So far no general expression for the energy of motion in terms of the field vectors has been obtained which will satisfy the condition that the total energy of motion in a moving electron's field is equal to the expression (14).

In a general electromagnetic field it is probably not permissible to regard the momentum and kinetic energy in an electromagnetic field as arising from the motion of a mass in the direction of Poynting's vector. To elucidate matters a little let us consider the result of superposing two fields $(-E, -H)$ and $(E + dE, H + dH)$ which differ very slightly and which are both simple radiant fields. Let the unit vectors s and $s + ds$ indicate the direction of Poynting's vector S in the two fields, then on account of the assumed property of the fields we have the relations

$$E + s \times H = 0, \qquad dE + s \times dH + ds \times H = 0,$$
$$s \cdot E = 0, \qquad s \cdot H = 0.$$

Since $s \cdot ds = 0$ we may write $ds = AE + BH$ and so

$$ds \times H = AE \times H = \lambda s;$$

this means that the vector s satisfies equations of type

$$dE + s \times dH = ks, \qquad s \cdot dE = k,$$

and so is one of the unit vectors (of the type considered in § 1) for the resultant field (dE, dH).

The energy $\frac{1}{2}(dE)^2 + \frac{1}{2}(dH)^2$ in the resultant field is generally very much less than the sum of the energies

$$\tfrac{1}{2}E^2 + \tfrac{1}{2}H^2, \qquad \tfrac{1}{2}(E + dE)^2 + \tfrac{1}{2}(H + dH)^2$$

[1] See for instance, L. Page, Amer. Jour. Sci., Vol. XL., Aug., 1915, p. 119. It should be mentioned that in the derivation of the formula there is an assumption with regard to the size of the electron when at rest as compared with its size when in motion.

in the component fields, so that there is a loss of energy of amount $E(E + dE) + H(H + dH)$ due to interference. This energy may perhaps be regarded as concealed energy of motion. The question now arises how two fields in which energy flows practically in the direction of the unit vector s can give a resultant field in which energy flows in a direction different from s. The answer is that the flow at right angles to s in the resultant field arises from the motion of a very large amount of energy with a very small component velocity perpendicular to s.

This may be the explanation of the perplexing fact that in the field of a moving electron the direction of Poynting's vector is quite different from that of the radius from the effective position of the electron. Indeed the field of a moving electron can be built up from simple radiant fields in each of which the flow of energy to a point is actually or very nearly along the radius just mentioned and it is just because the total amount of energy in these radiant fields is vastly greater than the energy usually attributed to the electron's field that the slight deviations in the paths from the mean radius can give rise to an appreciable transverse flow.

In our opinion then there is a colossal amount of concealed energy of motion in the field of an electron or positive nucleus of an atom. Whether this energy will ever become available or not we are quite unable to say. Of course it must be remembered that our theory of the structure of an electric field is based on the idea that the electric charge of an electron is continually being renewed[1] by electric separation (*i. e.*, the breaking up of minute doublets) and that the charge remains constant because a steady state has been reached. At present there is no way of deciding between this theory and the usual theory that an electron always consists of the same particles of electricity, but in support of the new theory it may be claimed that it gives a simple geometrical reason for the shape of the lines of electric force of a moving electric pole[2] and provides a possible explanation of gravitation as an effect due to an extremely slight fluctuation of the charges on electrons and positive nuclei in what may be slight deviations from the steady state of renewal of these charges.

The chief reason for pursuing a theory of this kind is the hope that it may throw some light on the nature of force and the real meaning of the dynamical equations of motion. It is very probable that the equations of motion are fundamentally of a geometrical nature[3] implying

[1] An idea somewhat similar to this is adopted by the late S. B. McLaren in his theory of gravitation, Phil. Mag., Vol. 26, 1913, p. 636.

[2] Mess. of Math., Vol. 47, 1918, p. 161.

[3] An attempt to express the fundamental laws of physical phenomena by geometrical considerations has been made recently by H. A. Lorentz, Amst. Proc., 19, pp. 1341–1369; 20,

the existence of certain incidences and correspondences with perhaps a minimum principle thrown in. While the general problem still baffles us some useful information may perhaps be gained by considering cases in which we can actually find an equivalent of the energy lost when work is done.

We shall commence by considering an electromagnetic field in which the usual equations

$$\begin{aligned} c \operatorname{rot} H &= \frac{\partial E}{\partial t} + \rho v, & \operatorname{div} E &= \rho, \\ c \operatorname{rot} E &= -\frac{\partial H}{\partial t} + \sigma w, & \operatorname{div} H &= -\sigma, \end{aligned} \tag{15}$$

are satisfied and as usual $\sigma = 0$, $\sigma w = 0$. We shall assume, however, that ρ and ρv have the forms

$$\rho = -\frac{1}{c}\frac{\partial \psi}{\partial t}, \qquad \rho v = c\nabla\psi. \tag{16}$$

The usual form of the energy equation is, moreover, obtained from the relation

$$\rho(v \cdot E) + \frac{\partial}{\partial t}[\tfrac{1}{2}(E^2 + H^2)] + \operatorname{div}[c(E \times H)] = 0, \tag{17}$$

where the first term represents the rate at which work is being done by the field on the electric charges present.

Now

$$c\operatorname{div}(\psi E) = -\psi\frac{\partial \psi}{\partial t} + \rho(v \cdot E),$$

hence the energy equation may be written in the form

$$\frac{\partial}{\partial t}[\tfrac{1}{2}(E^2 + H^2 + \psi^2)] + \operatorname{div}[c(E \times H) + c\psi E] = 0,$$

and the career of the energy which has been transformed into work is not lost sight of if we assume that the total amount of energy per unit volume is $\frac{1}{2}(E^2 + H^2 + \psi^2)$ and that the total flow of energy is specified by the vector

$$c(E \times H) + c\psi E.$$

The lack of symmetry with respect to E and H can be avoided if we write

$$\sigma = -\frac{1}{c}\frac{\partial \chi}{\partial t}, \qquad \sigma w = c\nabla\chi,$$

pp. 2–34, 1917. Further interesting geometrical developments may enter in the study of the growth of simple correspondences between different parts of the field figure and in estimating the closeness of fit of an imperfect correspondence.

and add the term $-(\sigma w \cdot H)$ to the left-hand side of (17) for then

$$c \operatorname{div}(\chi H) = \chi \frac{\partial \chi}{\partial t} + \sigma(w \cdot H),$$

and the energy equation may be written in the form

$$\frac{\partial}{\partial t}[\tfrac{1}{2}(E^2 + H^2 + \psi^2 + \chi^2)] + \operatorname{div}[c(E \times H) + c\psi E - c\chi H] = 0.$$

Similarly the usual momentum equations of type

$$\frac{\partial X_x}{\partial x} + \frac{\partial X_y}{\partial y} + \frac{\partial X_z}{\partial z} + \frac{1}{c}\frac{\partial X_t}{\partial t}$$

$$= \rho\left[E_x + \frac{1}{c}(v_y H_z - v_z H_y)\right] - \sigma\left[H_x - \frac{1}{c}(w_y E_z - w_z E_y)\right],$$

in which

$$X_x = E_x^2 + H_x^2 - \tfrac{1}{2}(E^2 + H^2), \qquad X_z = E_x E_z + H_x H_z,$$
$$X_y = E_x E_y + H_x H_y, \qquad X_t = E_z H_y - E_y H_z$$

may be written in the form

$$\frac{\partial X_x'}{\partial x} + \frac{\partial X_y'}{\partial y} + \frac{\partial X_z'}{\partial z} + \frac{1}{c}\frac{\partial X_t'}{\partial t} = 0,$$

where

$$X_x' = X_x + \tfrac{1}{2}(\psi^2 + \chi^2), \qquad X_z' = X_z + \psi H_y + \chi E_y,$$
$$X_y' = X_y - \psi H_z - \chi E_z, \qquad X_t' = X_t + \psi E_x - \chi H_x.$$

If we write

$$M = H + iE, \qquad -i\phi = \psi + i\chi,$$

the fundamental equations may be written in the form

$$c \operatorname{rot} M = -i\left(\frac{\partial M}{\partial t} + c\nabla\phi\right),$$
$$\operatorname{div} M + \frac{1}{c}\frac{\partial \phi}{\partial t} = 0, \tag{18}$$

and an interesting class of solutions is obtained by adding the additional equation[1] $M^2 = \phi^2$.

In this case we may write

$$M = \phi \frac{s_1 - s_2 + i(s_1 \times s_2)}{1 - (s_1 \cdot s_2)},$$

where s_1 and s_2 are unit vectors which are easily seen to satisfy equations (2) of § 1. Assuming that these are real and writing l for the x-com-

[1] Cf. H. Bateman, Mess. of Math., 47, 1918, p. 161.

ponent of s_2, X for the vector with components X_x, X_y, X_z, and K for the quantity

$$2\frac{\psi^2+\chi^2}{1-(s_1\cdot s_2)}$$

we have

$$\tfrac{1}{2}(E^2+H^2+\psi^2+\chi^2)=K,$$
$$\psi E-\chi H+E\times H=Ks_1,$$
$$\psi E-\chi H-E\times H=-Ks_2,$$
$$X=-Kls_1,$$
$$X_t=-Kl.$$

This means that the career of the energy transformed into work is not lost sight of if we suppose that an amount of energy K flows with the velocity of light in the direction of the unit vector s_1 and that momentum $-Ks_2$ flows with the velocity of light in the same direction. This momentum may perhaps be supposed to arise from the motion of particles in directions differing very slightly from the direction s_1, a condition which suggests the existence of two or more superposed fields which differ very slightly in properties. If a field of the present type is supposed to arise from the breaking up of minute electric and magnetic doublets and a rectilinear motion of their constituents with the velocity of light the dynamical equations of motion may simply imply that certain groups of particles travel along without losing any of their energy or momentum. In the general case, however, it is more probable that the dynamical laws tell us what happens when old groups of particles are broken up and new ones formed. It should be mentioned that a more general type of field has been found in which the career of the energy transformed into work at an ordinary point of space is not lost sight of. In this case we take

$$\rho v=c\nabla\alpha+c\lambda s_1,\qquad \rho=-\frac{1}{c}\frac{\partial\alpha}{\partial t}+\lambda,$$

$$\sigma w=c\nabla\beta+c\mu s_1,\qquad \sigma=-\frac{1}{c}\frac{\partial\beta}{\partial t}+\mu,$$

where s_1 is one of the unit vectors connected with the field by equations (2) of § 1. If now α and β are defined by the equations

$$\alpha=(s_1\cdot E),\qquad \beta=-(s_1\cdot H),$$

we have as before with α and β in place of ψ and χ

$$(\rho v\cdot E)=c\ \mathrm{div}\ (\alpha E)+\alpha\frac{\partial\alpha}{\partial t},$$

$$(\sigma w\cdot H)=c\ \mathrm{div}\ (\beta H)-\beta\frac{\partial\beta}{\partial t},$$

whatever λ and μ may be, but these must be chosen, of course, so that the equations of continuity

$$\frac{\partial \rho}{\partial t} + \operatorname{div}(\rho v) = 0, \qquad \frac{\partial \sigma}{\partial t} + \operatorname{div}(\sigma w) = 0$$

are satisfied. The energy in the field is now

$$T = \tfrac{1}{2}(E^2 + H^2 + \alpha^2 + \beta^2) = \tfrac{1}{2}(E^2 + H^2) + \tfrac{1}{2}[(E^2 - H^2)^2 + 4(E \cdot H)^2]^{1/2},$$

and the flow of energy

$$S = c[(E \times H) + E(s_1 \cdot E) + H(s_1 \cdot H)] = cTs_1.$$

On the other hand we have

$$X_t = [E_x(s_1 \cdot E) + H_x(s_1 \cdot H) - (E \times H)_x] = -T(s_2)_x,$$

where s_2 is the second unit vector connected with the field. We also have $X = -Ts_1(s_2)_x$, hence a total amount of energy T flows with the velocity of light in the direction of s_1 and a total momentum $-(1/c)Ts_2$ flows with the velocity of light in the same direction. It is probable that this type of field also arises from the breaking up of electric and magnetic doublets and the rectilinear motion of their constituents with the velocity of light. The above argument probably breaks down at points where the breaking up of the doublets occurs.

Returning to the previous type of field we note that equations (18) may be satisfied by writing

$$M = \frac{1}{c}\frac{\partial \Gamma}{\partial t} + i \operatorname{rot} \Gamma + \nabla \Phi,$$

$$\phi = -\operatorname{div} \Gamma - \frac{1}{c}\frac{\partial \Phi}{\partial t},$$

where Φ and the components of Γ are solutions of the wave equation. If $\chi = 0$ so that there is no real magnetism we have simply a field in which the volume density of electricity is proportional to the rate of change of a function ψ and the convection current is also derivable from ψ by differentiation. Let us call ψ the *electric storage*. It is important now to notice that *there may be a finite constant value of ψ and no electromagnetic field*, for if in the above equations we write

$$\Gamma = -\nabla \theta, \qquad \Phi = \frac{1}{c}\frac{\partial \theta}{\partial t},$$

we have clearly $M = 0$ and

$$\phi = \nabla^2 \theta - \frac{1}{c^2}\frac{\partial^2 \theta}{\partial t^2}.$$

It is only necessary then to choose θ to be a solution of the equation

$$\nabla^2\theta - \frac{1}{c^2}\frac{\partial^2\theta}{\partial t^2} = \text{constant},$$

in order to ensure that Φ and the components of Γ may be solutions of the wave-equation. The quantity ϕ is then constant and so therefore is ψ while there is no electromagnetic field.

This result indicates that in any field of type (18) we can regard the electric storage ψ as a positive quantity.

Electric storage is in some respects analogous to quantity of proto-electricity but the function ψ is an absolute invariant under the transformations of the theory of relativity. It seems more reasonable to regard quantity of proto-electricity as the rate of change of electric storage; the existence of both positive and negative charges of proto-electricity then seems quite natural.

Summary.

1. An electromagnetic field is studied geometrically in relation to a moving observer and various vectors are located with the aid of the two cones which at each point limit the directions of the forces acting on electric and magnetic charges moving with velocities less than that of light.

2. The electromagnetic laws are expressed directly in terms of the forces on unit electric and magnetic charges in motion and some deductions relating to lines of force are made from Hargreaves' theorems for space time integrals. One of Hargreaves' theorems suggests that quantity of electricity may represent a rate of change of another entity—quantity of proto-electricity, and electromagnetic fields are regarded as derivable from proto-electromagnetic fields containing line charges by a method analogous to differentiation.

3. Field vectors satisfying Maxwell's equations are constructed from the assumption that an aggregate of particles travel along straight lines with the velocity, and from a further hypothesis regarding the nature of the vectors.

4. A critical discussion is given on the nature of the energy in an electromagnetic field and the nature of its flow. A theory is developed which indicates that there is a colossal amount of concealed energy in the field of a moving electron.

Some examples are given which indicate what becomes of the energy which is apparently lost or transformed when work is done by an electromagnetic field on the electric charges within it.

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IONIZATION OF MERCURY, SODIUM AND POTASSIUM VAPORS AND THE PRODUCTION OF LOW VOLTAGE ARCS IN THESE VAPORS.

BY T. C. HEBB.

IT has been well established that when the vapors of a number of the metals are bombarded with electrons that there are at least two critical velocities of the electrons for each vapor: a lower velocity, or "resonance" velocity, produces the single-lined spectrum of the vapor under experimentation, and a higher velocity produces ionization of the vapor and a consequent emission of the complete spectrum. It has further been established that the equation

$$\tfrac{1}{2}mv^2 = h\nu$$

applies for these two velocities where ν, the frequency, assumes the values corresponding to the first and convergent wave-lengths of one of the optical series of the element.

These phenomena have been explained on the basis of Bohr's atom by assuming that at the "resonance" velocity an electron is knocked from its stationary orbit to the next outer orbit and that in falling back to its original position it radiates its surplus of energy with a frequency corresponding to the first member of the series belonging to this orbit; and that at the ionizing velocity the electron is knocked from its stationary orbit and is completely detached from the atom and in settling back to its normal position radiates the complete spectrum.

It is interesting to compare the results in the permanent gases, H, He, A and N, with those in the metal vapors. There is some evidence that what corresponds to the resonance velocity in vapors is what is found experimentally to be the ionization velocity in some of the gases. Thus for hydrogen the ionization potential instead of being 13.6 volts as called for by the Bohr theory or as might be suspected by analogy from the metal vapors is 10.2 volts which corresponds to the resonance potential in the vapors. That is, if in the equation $Ve = h\nu$ we introduce a value of ν corresponding to the first member of the Lyman series we get 10.2 volts, whereas if we use the value of ν corresponding to the last member, *i. e.*, the convergence frequency, we get 13.6 volts.

There is also some evidence that a similar condition holds for helium. Thus Richardson and Bazzoni[1] have shown that the limit of the helium spectrum lies between 470 A.U. and 420 A.U. If we take the mean of these two values, viz., 445 A.U., and substitute its frequency in the equation $Ve = h\nu$ we obtain a value of V equal to 28 volts. And if we assume that the first optical series of helium is given by the equation

$$\nu = k\left(\frac{1}{1^2} - \frac{1}{m^2}\right)$$

we see that the potential difference corresponding to the first member of the series is .75 of 28 = 21 volts. This is not very different from the observed value of the ionization potential.

The evidence offered by argon and nitrogen although small is worthy of consideration. Lyman[2] has shown that the spectrum of argon ends in the vicinity of 800 A.U. and that of nitrogen in the vicinity of 975 A.U. If the corresponding frequencies be introduced in the equation $Ve = h\nu$ we get 15.5 volts and 12.4 volts respectively. The ionizing potentials for these two elements are 12 volts and 7.5 volts respectively and it is significant that in the case of each element the observed ionization potential is less than that which would be expected from a knowledge of the high frequency limit of its spectrum and an application of the quantum relation $Ve = h\nu$.

An endeavor has been made to explain the low ionization potentials of helium[3] and hydrogen by assuming that as soon as radiation is produced by the collisions of electrons with atoms, this radiation acts photo-electrically on the cathode and liberates electrons with sufficient initial velocity to account for the difference between the observed and theoretical values of the ionization potential. According to the Bohr theory radiation would first occur when the electron is knocked from its stationary orbit to the one next outside of it.

The distinction between the two groups, the vapors and the gases, does not seem to be as rigid as outlined above for it has been shown that under certain conditions mercury vapor[4] is ionized at its resonant potential. And this suggests that possibly the other vapors are capable of a similar action. This paper is a report of some experiments I have performed on potassium and sodium vapors.

For various reasons these elements were studied in an atmosphere of mercury vapor. Although I began the experiments partly with the idea

[1] Richardson and Bazzoni, Phil. Mag., 6, 34, 285, 1917.

[2] Lyman, Astrophysical Journal, 43, 89, 1916.

[3] Van der Bijl, PHYS. REV., 2, 10, 546, 1917.

[4] Hebb, PHYS. REV., 2, 9, 371, 1917 and 2, 11, 170, 1918.

of determining the effects sodium and potassium radiations would have on the mercury vapor I would have extended the experiments to the case of pure sodium and potassium vapors if I had been fortunate in procuring a continuous action low vacuum pump. As the pump was not available that phase of the experiment cannot be reported.

The experimental tube was of the same nature as that used in the study of mercury vapor. It consisted of a glass tube about 2.5 cm. in diameter and 20 cm. in length bent at the center so that when placed horizontally with the bend down a supply of mercury always remained in this depression. The platinum cathode—width 4 mm., length 1.0 cm., thickness .0025 cm.—containing a small patch of CaO was supported at the center of the tube by two iron lead wires. The anode, always capped with platinum, was placed 3 to 4 mm. from the cathode. The ends of the tube were sealed with Khotinsky cement and kept cool by water jackets. The central part of the tube was heated by a gas-heated furnace. The apparatus was connected to a drying tube, a McLeod gauge and the pump. The latter was capable of producing a vacuum of .005 cm. Of course a very pure atmosphere of potassium or sodium and mercury vapors could be obtained by simply heating the central part of the tube. As the vapor pressures of potassium and sodium are small at the temperatures necessarily used with mercury, the major part of the vapor surrounding the anode and cathode was that of mercury. Neither the sodium nor the potassium was distilled before using. It was considered that the necessary purification would take place in the experimental tube and for this reason the mercury was boiled for some time, with the pump running, before observations were made.

The metals, sodium and potassium, were sometimes introduced into the mercury before placing them in the tube and sometimes were placed

Fig. 1.

pure in a receptacle at the end of the anode. This needs a little explanation. In order to get a greater density of the sodium or potassium vapor it was found necessary to use an anode as shown in Fig. 1.

It was made of glass with a chamber *C* sealed off at the point *B*. A platinum wire passed through the anode and made contact with a removable platinum cap *A*. In the center of *A* and opposite the center of the cathode there was a small hole. The sodium or potassium was placed in the chamber *C* and, the platinum cap having been adjusted, the anode was quickly placed into the experimental tube. The mercury

was then boiled for some time with the pump running in order to remove impurities. Then when the cathode was heated in close proximity to the cap *A* the sodium or potassium vapor was forced out through the small hole in *A* and was very favorably situated with respect to ionization.

Results with Potassium.

It was found that the striking voltage of the arc was lower than the ionization potential, 4.1 volts, and that with favorable density of the potassium vapor it was possible to get the arc to strike as low as 1.5 volts. When the density of the potassium vapor was small, which was usually the case when the potassium was simply dissolved in the mercury and none was placed in the special anode, the arc did not strike as low as when the density was greater nor did the spectrum of potassium usually appear. This was true for potential difference as low as 2.0 volts in rare cases and commonly for potential differences of 3.0 volts or higher. It should be stated that the spectra were observed with the aid of a small direct vision spectroscope so that it is not claimed that the potassium lines were not present but relatively to the mercury lines they must

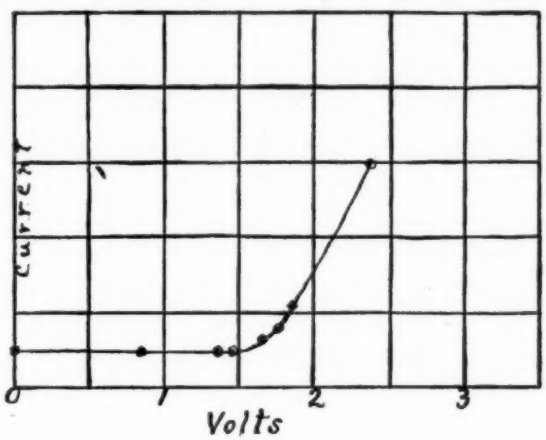


Fig. 2.

have been very weak. This apparently means that the current was carried almost entirely by the mercury vapor. When, however, the arc struck at potential differences lower than 2.0 volts the spectrum of potassium always appeared in strength. But even at these low voltages the mercury spectrum always appeared too.

As I had a low-resistance galvanometer in series with the arc I could make observations on the relation between the current through, and the potential across the arc space. Fig. 2 shows such a current-potential curve. It will be noticed that the curve starts to bend at about 1.5

volts. In this case the potassium had been dissolved in the mercury before introducing it into the tube. A simple anode terminating in a piece of platinum foil was used. At about 2.4 volts the mercury spectrum was showing but no potassium lines could be seen.

I have previously shown that the mercury arc could be made to operate at a potential difference lower than its striking voltage.[1] I have found that in the case of potassium the same thing is true. After the arc struck the voltage nearly always dropped as in the case of mercury and by manipulating the potential across the arc I have had it operating as low as 0.5 volts. It is possible that it could be obtained lower, as no special effort was made to find its minimum value. But a very remarkable thing was that the mercury spectrum still showed at 0.5 volts.

RESULTS WITH SODIUM.

As the melting point of sodium is considerably higher than that of potassium and as its vapor pressure is much lower at the temperatures I was using it did not lend itself well to experimenting in a mercury atmosphere. However, it caused the striking voltage of the arc to fall but not so pronouncedly as in the case of potassium. The lowest I have had the arc strike has been about 2.5 volts, but 3.0 to 3.5 volts was more common. The yellow lines of sodium appeared, but as in the case of potassium the rest of the spectrum was frequently not present. I have not been able to obtain a current-potential curve which broke as low as 2.1 volts—the resonance potential. I do not doubt but that it can be obtained.

Wood and Okano[2] have shown that the spectrum of sodium can be produced at 2.3 volts and that the *D* lines could be obtained with a potential difference as small as 0.5 volts. On several occasions I have had the *D* lines at potential differences lower than 1.0 volt. From a study of the current potential curves for sodium I am of the opinion that the appearance of these lines is not due to ionization. It is of interest to note that the occurrence of the *D* lines below 2.1 volts is similar to the appearance of the single line of mercury below 4.9 volts.[5]

As in the case of mercury and potassium, it was found possible to get the arc to operate at a voltage lower than its striking value and lower than the resonance potential of sodium. In the case of sodium I have had the arc operate as low as 1.4 volts with both the sodium and mercury lines showing.

DISCUSSION OF RESULTS.

These results prove that in the case of potassium and probably in the case of sodium ionization can occur at a voltage V given by the equation

[1] Hebb, PHYS. REV., 2, 9, 371, 1917.

[2] Wood and Okano, Phil. Mag., 6, 34, 177, 1917.

$Ve = h\nu$ where ν is the frequency of the first member of the principal series of the element. These elements thus fall in line with mercury and more or less with the cases of hydrogen and helium. The results also appear to throw light on the cause of ionization at the resonance voltage. In the case of mercury it has been suggested that it is the radiation $\lambda = 2{,}536$ A.U. produced by 4.9 volt electrons, which acting photo-electrically causes the ionization. In the cases of sodium and potassium the radiation from the hot cathode should have caused the ionization if the radiations $\lambda = 5{,}890$ for sodium, and $\lambda = 7{,}700$ for potassium were all that was necessary. It thus seems to be certain that the ionizing electron must travel with a velocity corresponding to a fall through its resonant voltage at least before ionization can occur. This does not prove that the ionization is due entirely to collision, but it does seem to prove that collision is necessary and that there is a certain minimum velocity for that collision. That the radiation seems to have a small effect was indicated by former experiments with mercury. Similar experiments were tried with potassium. The light from an arc—the carbons of which had been saturated in potassium chloride—was passed into the experimental tube. No effect could be observed on the ionization potential or on the current through the arc. The galvanometer used was sensitive to about 10^{-5} amperes.

There are several ways by which we might imagine the ionization at the resonance voltage to occur: (1) It may be due to repeated collisions, (2) it may be due to fast electrons liberated from the cathode photo-electrically, (3) it may be due to photo-electric action on the vapor and (4) it may be due to a combination of the above. Cases 2 and 3 seem to be entirely ruled out for sodium, potassium and mercury but as Van der Bijl[3] has pointed out 2 might be effective in the case of helium, and he might have added hydrogen, for 10.2 volt light should liberate electrons from tungsten or platinum with a velocity of approximately 6.0 volts. In a similar manner we can imagine that argon is ionized at 12 volts, for if we take the lower limit of its spectrum to be 800 A.U.—15.5 equivalent volts—it would only mean a difference of 3.5 volts between observed and expected ionization potential. The lower limit of nitrogen's spectrum is 975 A.U. and this corresponds to a voltage of 12.7. If we take the results of Goucher wihch were apparently obtained with a pure platinum cathode, the long wave-length limit of which is 2910 A.U.—equivalent to 4.3 volts—we find that 7.4 volt light would only produce electrons with an initial velocity of 3.0 volts. This added to 7.4 only gives 10.5 volts which would not be sufficient to produce ionization.[1]

[1] Of course the low ionization potential of argon and nitrogen might be due to the radiation acting on the anode.

Thus while the photo-electric action on the cathode can explain the low ionization for some of the elements it cannot do so for the others. And as pointed out above the photo-electric action on the vapor under experiment does not appear to be able to explain the results. Hence it appears as if the ionization at the resonance voltage was due either to repeated collisions or to a combination of collision and photo-electric effect, but that in either case a minimum velocity of collision was necessary, this minimum velocity corresponding to the resonant voltage. That the single-lined radiation is not able to ionize the vapor under experiment to any appreciable extent is also indicated by the fact that a strong single-lined radiation can be obtained both in mercury and sodium vapor below the resonance voltage, as has been shown by the writer for mercury[5] and by Wood and Okano[6] and the writer for sodium.

Although it requires a minimum potential difference equal to the resonance voltage to start appreciable ionization in the vapors of mercury, sodium and potassium, it does not require this voltage to maintain an arc in these vapors. Both in these experiments and in previous experiments I have had the arc in pure mercury vapor operating as low as 3.0 volts. I have also had the sodium arc operating as low as 1.4 volts and the potassium arc as low as 0.5 volts. Since getting the above results for mercury they have been substantiated by McLennan[1] who states that he has also had a cadmium arc to operate between 2 and 3 volts. As the resonance potential of cadmium is 3.8 volts this is another case where the arc operates lower than the resonance voltage of the vapor. Apparently the same phenomena hold for argon as MacKay and Ferguson[2] state that they have had an arc operating in argon at 4.0 volts. If we consider 12 volts the resonance potential of argon this is considerably below that value.

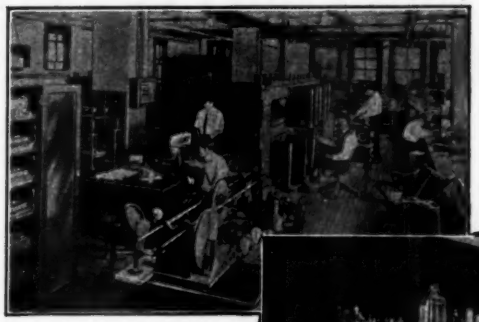
If we attempt to explain these low arcs on the basis of the photo-electric action of the arc radiation on the cathode, it would appear from the following table that the velocities of emission are sufficiently great with the possible exception of potassium.

Vapor.	Ionization Potential.	Resonance Potential.	Material of Cathode.	Long Wave-length in Eq. Volts.	Max. Vel. in Eq. Volts.
Hg	10.3	4.9	CaO	3.4	6.9
Na	5.1	2.1	"	"	1.7
K	4.1	1.6	"	"	.7
Cd	8.9	3.8	W	4.5	4.3
A	15.5?	12?	"	"	11.0

[1] McLennan, PHYS. REV., 2, 10, 84, 1917.

[2] MacKay and Ferguson, PHYS. REV., 2, 7, 410, 1916.

Opportunity



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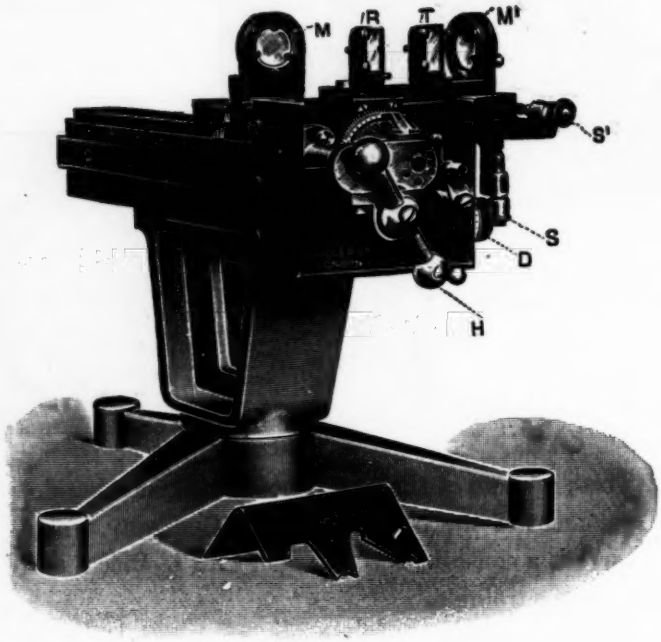

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However, when we consider the magnitude of the arc currrents it does not seem as if the photo-electric action alone was sufficient. Thus in one case with pure mercury vapor the thermionic current was 6×10^{-5} amperes before ionization began at 5.0 volts. After the arc had struck and was operating at 3.7 volts the current was 1800×10^{-5} amperes. After making allowance for the increased electron emission due to the hot point on the cathode this appears to demand entirely too high a photo-electric current from a small cathode.

But still other difficulties exist. It has been shown that an arc will strike as low as about 2.0 volts in an atmosphere of potassium and mercury vapor, and that only the mercury lines are evident. If we assume that some of the potassium vapor ionizes and that the consequent radiation acts on the CaO the 4.1-volt light would liberate electrons from the cathode with a velocity of about 0.7 volts. This added to 2.0 volts would still be insufficient to ionize the mercury vapor. Nor would the action of the 4.1-volt light have any photo-electric effect on the mercury vapor as the long wave-length limit of mercury itself is about 2,800 A.U. The difficulty is still more pronounced in the case where the potassium-mercury arc struck at 1.5 volts and operated as low as 0.5 volts with the mercury spectrum showing. There is, however, the possibility that sufficient potassium adheres to the cool ends of the cathode so that when the potassium vapor ionizes its radiation acts on the solid or liquid potassium and liberates electrons with sufficient velocity to ionize the mercury vapor. The velocity of emission can easily be imagined to be of the right magnitude.

From these experiments and in view of the above considerations it appears to be altogether likely that there are other means of producing ionization and light in these arcs than those considered above. It is possible that the action of the positives is not sufficiently considered and that impacts of the positives on the cathode liberate electrons with an initial velocity as claimed by Tate.[1] Further it seems altogether probable that some of the light of the arc is produced by the positives as Dempster[2] has shown that positives moving with velocities less than that corresponding to 5.0 volts are able to stimulate line spectra. And of course the possibility of chemical action, especially in the case where two metals are employed, is to be considered.

SUMMARY.

It has been shown that: (1) Potassium vapor can be ionized at 1.6 volts, its resonance voltage. (2) Sodium vapor can be ionized at 2.5

[1] Tate, PHYS. REV., 2, 10, 81, 1917.

[2] Dempster, PHYS. REV., 2, 8, 651, 1916.

volts which is very close to the resonance voltage. This result agrees with that found by Wood and Okano. (3) The *D* lines of sodium can be excited at less than 1.0 volt; also in agreement with Wood and Okano. (4) The sodium and potassium arcs in mercury vapor can operate below their resonance potentials and as low as 1.4 volts for sodium and 0.5 volts for potassium. (5) The mercury spectrum can be produced as low as 0.5 volts in an atmosphere of mercury and potassium.

UNIVERSITY OF BRITISH COLUMBIA,
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PROCEEDINGS

OF THE

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A POSSIBLE STANDARD OF SOUND.[1]

BY CHAS. T. KNIPP.

THE paper as presented described a source of sound recently brought to the writer's attention, while blowing a mercury vapor trap of pyrex glass, that bids fair to furnish a standard of sound of any desired pitch with no other apparatus than the trap and a bunsen burner. In its simplest form the apparatus is an ordinary trap as shown in Fig. 1, having the usual ring seal at *M*.

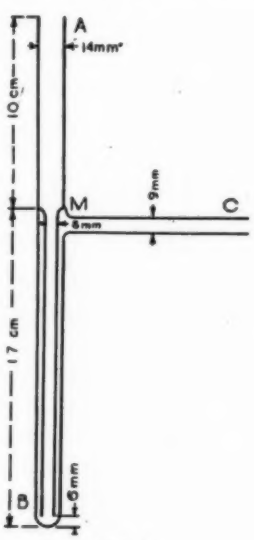

Fig. 1.

To operate, close *A* with a sliding piston of cork, let *C* remain open, and apply a bunsen burner (adjusted to give a fairly hot flame) at *B*. The tube *AB* should be held in the flame at an angle so that the central portion *M* is not unduly heated. When *B* begins to glow a pure tone that is readily audible over a large room is emitted at *C*. The pitch of the sound is dependent upon the length of the vibrating column *AB* and also upon the length of the side tube *MC*. Attaching a horn at *C* intensifies the sound many fold. The only opening is at *C*, yet a candle placed at this point is instantly blown out. On closer examination it was noticed that a current of air *enters* the tube *C* around its edge, and another at the same time *escapes from it* along its axis.

There are other conditions that effect the pitch. Those noted thus far are: That heating the region about *M* destroys the sound, but on the other hand if the flame is removed from *B*, then *C* stopped and *A* opened the tube will again operate on *heating M to redness*. That the pitch is raised by the addi-

[1] Abstract of a paper presented at the Pittsburgh meeting of the American Physical Society, December 27–29, 1917. (This and the following abstract were duly submitted by the author prior to the meeting, but were delayed or went astray in subsequent transmittal to the review.—ED.)

tion of extra side tubes fused to the vibrating column at M, and is instantly lowered when these extra branches are in turn stopped.

Tubes having different dimensions were constructed. These can be adjusted over wide ranges—each an octave or more—and all give, apparently, clear tones particularly free from over tones. By supplying heat to the end B at a constant rate (as by an electric furnace) the pitch may be kept constant for an indefinite length of time. The apparatus should therefore furnish a standard source of sound.

LABORATORY OF PHYSICS,
UNIVERSITY OF ILLINOIS,
December, 1917.

AN IMPROVED FORM OF MERCURY VAPOR AIR PUMP.[1]

BY CHAS. T. KNIPP.

THE mercury vapor pump described in this paper retains the same simple valve arrangement described recently by the writer,[2] but on the other hand replaces the umbrella that deflected the mercury vapor downward through an annular throat by the commonly used aspirator nozzle through which the vapor issues vertically upwards. This necessitates an interchange of connections leading to the supporting pump and the vessel to be exhausted.

An improved form of mercury vapor pump.

This pump, single stage, will operate on any oil supporting pump of the grade of the Nelson pump. In addition to its speed, its simplicity of design and ease of construction are important points, and when constructed of pyrex glass is durable.

The paper also gives the data obtained when several of these pumps are placed in tandem. Again, a three stage pump retaining the same general principle is described, designed to operate on a poorly working water aspirator as a supporting pump. The mercury vapor for each stage is supplied from the same boiler, yet at different pressures—the highest pressure to the first stage exhausting into the aspirator. Sample pumps and sketches were exhibited.

LABORATORY OF PHYSICS,
UNIVERSITY OF ILLINOIS,
December, 1917.

[1] Abstract of a paper presented at the Pittsburgh meeting of the American Physical Society, December 27–29, 1917.

[2] PHYS. REV., N. S., IX., No. 3, March, 1917.

ERRATA.

Vol. XI., June, 1918, page 501, abstract by Irwin G. Priest entitled "The Law of Symmetry of the Visibility Function," third line from bottom of page, for $e^{-0.0002664(f-541)2}$ read $e^{-0.0002664(f-541)^2}$.

Vol. XI., June, 1918, page 504, abstract by Irwin G. Priest entitled "A Precision Method for Producing Artificial Daylight," in caption under Fig. 1, for "0.500 thick" read "0.500 mm. thick."

Vol. XII., October, 1918, in the article by James E. Ives, entitled "On the Effect of a Transverse Magnetic Field on the Discharge through a Geissler Tube," lines 3–9 inclusive on page 301 should be transferred bodily and inserted after line 3 on page 304. On page 303, the words "Field in Gausses" (indicating abscissae) should be above rather than below "Fig. 7."

Vol. XII., Oct., 1918, article by W. E. Tisdale, entitled "The Effects of Gases and Metallic Vapors on the Electrical Properties Exhibited by Selenium Crystals of the Hexagonal System," page 326, caption for Fig. 1*a* should be "Action curve for test crystals," caption for Fig. 1 should be "Diagram of apparatus"; page 328, line 6 from bottom of page, for word "positive" read "comparative."

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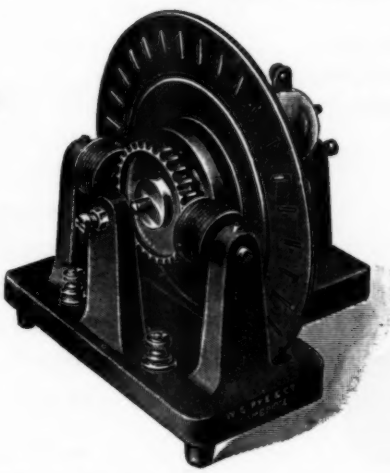

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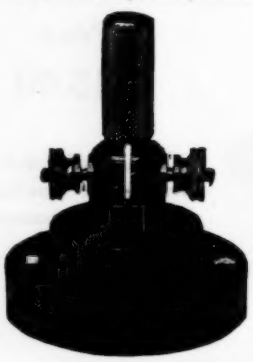

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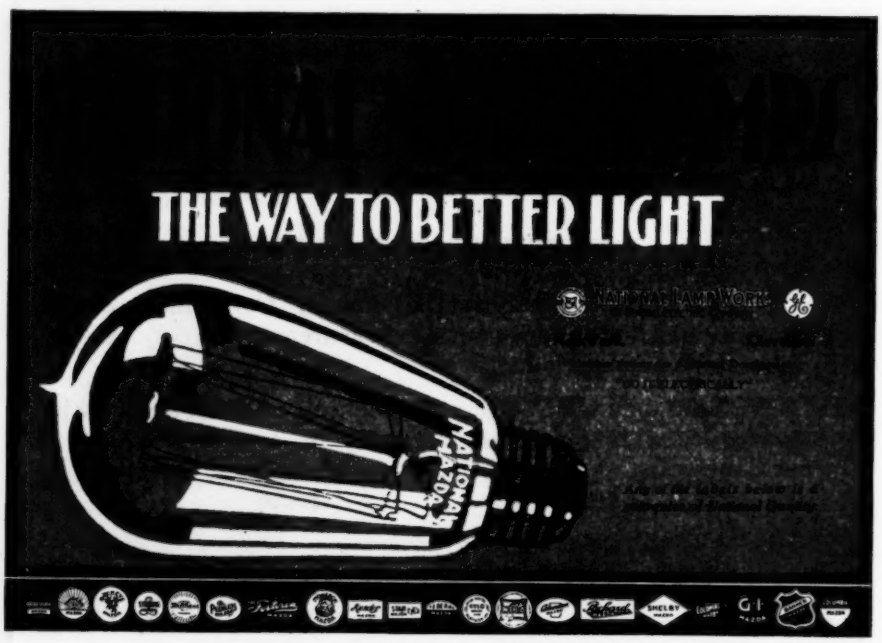

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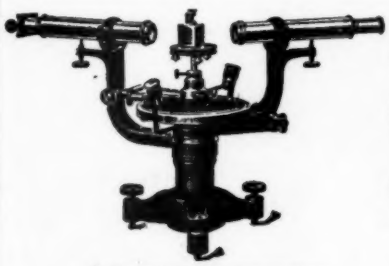

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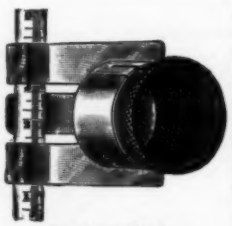

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The Langmuir Condensation Pump

Dr. Irving Langmuir (of the General Electric Company) has developed an exceedingly interesting and valuable *high-speed, high-vacuum pump;* and by special arrangement with the makers, we are acting as sole distributors of this device for laboratory purposes.

As shown in accompanying illustration, the pump is manufactured entirely of metal; and thus is far more permanent in character than would be possible if glass were used.

Connections are provided for water cooling; and beneath the pump is a small electric heater (terminals not shown in picture) for attachment to 110 volt lighting circuit.

With the Langmuir Pump pressures as low as 10^{-6} bar have been obtained, and there is little doubt that very much lower pressures can be produced by cooling the bulb to be exhausted, in liquid air, so as to decrease the rate at which gases escape from the walls.

As is known to physicists, one "bar" is equal to 0.00075 mm. of mercury.

Some type of primary pump must be used; capable of developing vacuum not less than 0.1–0.15 mm. of mercury.

In the December 1916 issue of General Electric Review, Dr. Langmuir published a paper entitled—"The Condensation Pump, an Improved Form of High-Vacuum Pump." It should be read by all scientists who are interested in high-vacuum work. Write for a copy.

$90.00 is the net price for a Langmuir Condensation Pump

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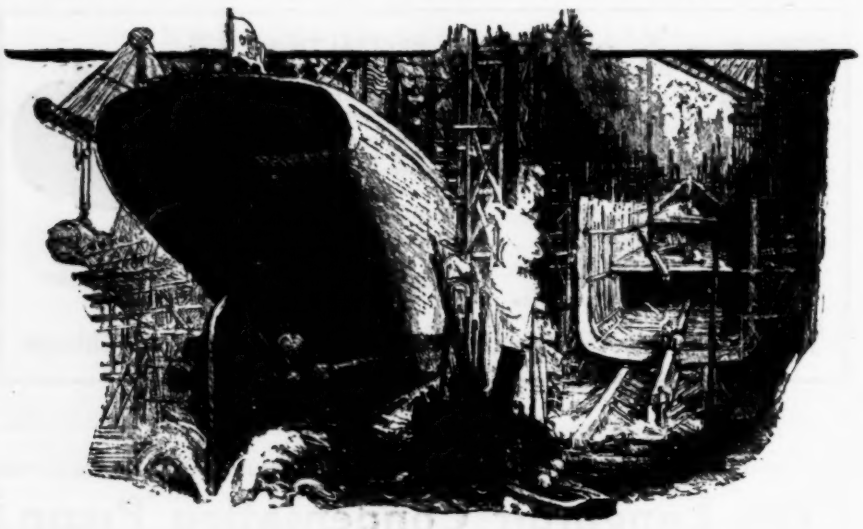

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Lumber for ships is not the whole story. The call for more cantonments and barracks, spruce for aeroplanes, workmen's houses and additional shipways comes at the same time. Powerful cranes and hoists, necessary in both steel and wooden ship construction, must also have electric power equipment.

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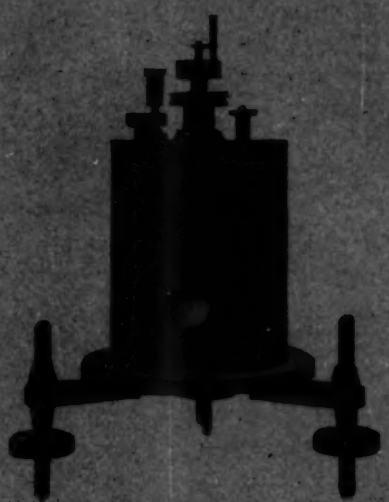

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